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# Airport congestion pricing when airlines price discriminate

Achim I. Czerny [a,*], Anming Zhang [b,c]

[a] WHU – Otto Beisheim School of Management, Germany
[b] University of British Columbia, Canada
[c] China Academy of Financial Research, Shanghai Jiao Tong University, China

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ABSTRACT

This paper extends the literature on airport congestion pricing by allowing carriers to price-discriminate between the business and leisure passengers when operating costs are the same for all passengers. The main results are: First, the second-best discriminating business fare exceeds the first-best uniform fare (which equals the external part of the marginal congestion costs), while the second-best discriminating leisure fare is lower than the first-best uniform fare. Second, the optimal airport charge implements the first-best uniform or second-best discriminating fares. Importantly, this charge can always be higher than what would be expected when all passengers were treated as having the same time valuation. This result provides some support to the finding that the welfare losses associated with an atomistic airport congestion charge may be low.



## 1. Introduction

For most parts of the last decade, air travel delays have been a major problem in many countries.[1] Ball et al. (2010) studied the economic impact of air travel delays in the US, and found that the cost of flight delays in 2007 is $31.2 billion. While the causes of delays can vary from country to country, the volume of traffic relative to airport capacity (mainly runways) is a major cause. In effect, the US Department of Transport identified airport congestion reduction as its No. 2 top management challenge, only second to aviation safety (USDOT, 2008c).

To manage airport congestion, economists have advocated the use of price mechanism, under which landing fees are based on a flight's contribution to congestion.[2] The early congestion-pricing models by, for example, Levine (1969), Carlin and Park (1970) and Borins (1978) were developed along a line similarly to dealing with road congestion. As such, flights (individual drivers) were treated as atomistic. The more recent literature recognized that the "atomistic" assumption may not hold for flights, since a congested airport is usually dominated by a few carriers, each of which runs a large number of flights at the airport and has market power. With the non-atomistic assumption the literature showed that carriers may themselves internalize congestion, and so the welfare-optimal airport charge should be reduced relative to the level where carriers were treated as atomistic (e.g., Daniel, 1995; Brueckner, 2002; Pels and Verhoef, 2004; Zhang and Zhang, 2006; Basso, 2008; Silva and Verhoef, 2013). Essentially, unlike each individual road driver who does not take external congestion (the congestion she imposes on other drivers) into account in her driving decision, a large airline might, in its flight decision, take into account

* Corresponding author. Tel.: +49 2616509223.
*E-mail addresses:* achim.czerny@whu.edu (A.I. Czerny), anming.zhang@sauder.ubc.ca (A. Zhang).

[1] Twenty percent of airline flights in the United States were delayed between 2000 and 2007. (A flight is considered as delayed when the actual arrival time exceeds the scheduled arrival time by more than 15 min.) Similar delays have plagued European and Asian airlines and airports. In China (the world's second largest air transport market behind the US) for example, more than thirty percent of its domestic flights were delayed in recent years.

[2] The US Department of Transport has, since 2008, allowed US airports to charge peak-period landing fees in addition to weight-based fees (USDOT, 2008a,b).

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the fact that scheduling one more flight generates extra congestion costs to its other flights and its passengers. Further, the larger a carrier's market share is, the greater it internalizes such flight congestion, leading to an inverse relationship between the optimal airport charge and market concentration (carrier market shares).

While these studies concentrate on uniform time valuations by passengers (and uniform airline fares), Czerny and Zhang (2011) and Yuen and Zhang (2011) recognize that passengers may have different time valuations. Particularly, Czerny and Zhang consider two passenger types: business passengers and leisure passengers, with business passengers exhibiting a high time valuation relative to leisure passengers.[3] They derive the welfare-optimal airport charge in this environment for uniform airfares.[4] A major insight of their analysis is that the incentive to internalize self-imposed congestion may be too low from the social viewpoint because the carriers are concerned with the "marginal" passenger's time valuation (i.e., the average time valuation of incremental passengers) rather than the average time valuation.[5] Importantly, this implies that the optimal airport charge can be higher than what would be expected when all passengers are assumed to have the same time valuation. Basically, an increase in the airport charge can improve welfare by protecting the high-time-value passengers from excessive congestion caused by the low-time-value passengers.

The literature on airport congestion pricing has so far assumed away the possibility that airlines may engage in price discrimination. Airlines nonetheless are a frequently used example for markets where price discrimination is prevalent (e.g., Borenstein, 1985; Dana, 1999a,b; Cowan, 2007). In a recent study, Lazarev (2013) found that leisure passengers start searching for a ticket at least six weeks prior to flight departure, while business passengers typically search in the last week. Thus airlines can use advanced-purchase rebates to price-discriminate between the business and leisure passengers, and charge business passengers a high fare relative to leisure passengers (e.g., Stavins, 2001; Hazledine, 2006). Czerny and Zhang (2014) developed a model that captures carrier third-degree price discrimination between the business and leisure passengers when their demands are interdependent because of airport congestion. They pointed out that a uniform fare is needed to implement the first-best solution. The economic intuition is that a passengers's congestion effect on all the other passengers is independent of her own time valuation. Consequently, the congestion externality to be internalized by fares is independent of the passenger type.

The present paper derives the socially optimal airport charge when airline price discrimination is allowed and all markets are covered. It further compares the first-best outcomes (under uniform pricing) with the second-best outcomes when carriers price-discriminate between the business and leisure passengers.[6] To accomplish these objectives, Czerny and Zhang's (2014) model, which concentrates on a monopoly airline, is extended in order to analyze the following two-stage game: the airport chooses its charge to maximize welfare in the first stage. In the second stage carriers compete, in a Cournot fashion, in the business-passengers and leisure-passengers markets subject to a *price-difference constraint*. This constraint implies that business passengers are charged with an exogenous premium on the leisure fare, and a simple comparative-static analysis between business and passenger fares in the premium added to the leisure fare can be used to identify the effect of price discrimination on business and leisure fares relative to the uniform fare. While the price-difference constraint is commonly used to compare the pricing behavior under uniform pricing and third-degree price discrimination (e.g., Leontief, 1940; Schmalensee, 1981; Varian, 1985; Holmes, 1989; Aguirre et al., 2010), to our best knowledge it has yet been used in a framework where firms compete in a Cournot fashion.

Two main insights are derived from this analysis: First, the second-best discriminating business fare exceeds the first-best uniform fare, while the second-best discriminating leisure fare is below the first-best uniform fare. Second, the second-best fares can be implemented by the right choice of the airport charge, while carrier price discrimination has no direct effect on the structure of the optimal airport charge. This second result is true in the sense that the effect of the time-valuation difference on self-internalization and thus the optimal airport charge is largely independent of whether carriers engage in price discrimination or just charge uniform fares. With or without price discrimination, therefore, an increase in the airport charge can improve welfare by protecting the high-time-value passengers from excessive congestion caused by the low-time-value passengers. The result is important as it shows that the relationship between airport charges and time valuations found in Czerny and Zhang (2011)—who abstracted away airline price discrimination—is robust with respect to the carriers' pricing behavior. Furthermore, this result provides some support to Daniel (1995, 2001, 2011), Daniel and Pahwa (2000), Daniel and Harback (2009) and Morrison and Winston (2007), who find that the welfare losses may be low if policy makers just implement the atomistic airport charge (i.e., the optimal airport charge that would be chosen as if carriers were atomistic).

The paper is organized as follows. Section 2 presents the main model specifications. Section 3 analyzes the social maximizer's pricing behavior. Specifically, this section introduces the price-difference constraint in order to analyze the relationship between the first-best uniform and second-best discriminating fares. The carrier's equilibrium pricing behavior

[3] For empirical evidence of this time-valuation difference, see e.g. Morrison (1987), Morrison and Winston (1989), USDOT (1997) and Pels et al. (2003).

[4] Yuen and Zhang (2011) assume that time valuations are positively correlated with the passengers' willigness to pay and consider the peak and off-peak periods, while Czerny and Zhang (2011) concentrate on a static framework. In addition, Daniel (2001) captures that time valuations may be different between aircraft.

[5] Note that the marginal passenger's time valuation refers to the average time valuation of incremental passengers. Since delays can be considered as a quality dimension for passengers, this is consistent with the analysis by Spence (1975) and Sheshinski (1976), which shows that monopoly suppliers are concerned with the marginal consumer's quality valuation, while the social maximizer is concerned with the average quality valuation (Zhang and Czerny, 2012).

[6] Second-best congestion pricing for roads has been investigated by, for example, Verhoef et al. (1996).

for a given airport charge is analyzed in Section 4. This section distinguishes between uniform pricing and laissez faire, where laissez faire refers to a scenario where the price-difference constraint is non-binding. Section 5 characterizes the welfare-optimal airport charge for a given price difference between business and leisure fares, and discusses analytical as well as numerical examples based on specific functional forms. Section 6 contains concluding remarks and discusses avenues for future research.

## 2. The model

Consider an origin–destination air travel market. Passengers are partitioned in two groups: business passengers and leisure passengers. Let $q_B$ denote the quantity of business passengers (to be referred to simply as the *business quantity*) and $q_L$ the leisure quantity. Setting aside congestion, passenger utilities (gross benefits from travel) are $B_B(q_B)$ in the business market and $B_L(q_L)$ in the leisure market. The utilities are strictly concave, i.e. $B_x' > 0$ and $B_x'' < 0$ for $x = B, L$.

There is a single, public airport with limited capacity supply that charges a per-passenger fee $\tau$, to be referred to as the airport charge, to carriers (which is without loss of generality). The limited capacity causes airport congestion and flight delays, and passengers incur congestion delays as a result. Following Brueckner (2002) and others, per-passenger delays $C$ depend on aggregate passenger quantity $q \equiv q_B + q_L$, with $C'(q) > 0$ and $C''(q) \geqslant 0$. Thus the per-passenger delays increase in the aggregate passenger quantity at a non-decreasing rate. How much the delays cost passengers depend on their values of time. Denoting the business passengers' time value by $v_B$ and the leisure passengers' by $v_L$, we consider cases where $v_B \geqslant v_L > 0$. Consequently, for given $q$ the delay costs are $v_B C(q)$ for business passengers and $v_L C(q)$ for leisure passengers.

Customers are served by $n$ identical carriers. Let $q_{Bi}$ and $q_{Li}$ denote carrier $i$'s business and leisure quantities, respectively, for $i = 1, 2, \ldots, n$ and $q_x \equiv \sum_i q_{xi}$. With the delays being the same for all passengers, there is a "generalized price" of traveling,

$$\eta_x \equiv p_x + v_x C, \quad x = B, L, \tag{1}$$

where $p_B$ denotes the fare charged to business passengers (to be referred to simply as the *business fare*) and $p_L$ the leisure fare. At the demand equilibrium the passengers, taking individual delays as given, equate their marginal utilities with the generalized prices (i.e., $B_x' = \eta_x$) leading to the following inverse demands:

$$P_x \equiv B_x' - v_x C, \quad x = B, L \tag{2}$$

with $p_x = P_x$. Since $\partial P_x / \partial q_y = -v_x C' < 0$, the (inverse) demands in (2) are interdependent.[7] The intuition behind this interdependency is clear: An increase in one market's quantity increases congestion for all passengers and, thus, the other market's generalized price.

## 3. Social maximizer's pricing behavior

This section assumes that there is a social maximizer who can directly choose uniform fares or discriminating business and leisure fares. This analysis provides a useful benchmark for the carriers' and the airport's pricing behavior analyzed in the subsequent parts of this study.

### 3.1. First-best uniform fares

It is convenient to let $v$ denote the business time value (i.e., $v \equiv v_B$) and $\alpha v$ the leisure time value, with $\alpha \in [0, 1]$ (i.e., $\alpha v \equiv v_L$). Assuming for simplicity zero operating costs for the airport and carriers (which helps to concentrate on the effect of demand elasticities on fares), social welfare can be written as

$$W \equiv B_B + B_L - q \bar{v} C, \tag{3}$$

where $\bar{v} \equiv (q_B + \alpha q_L) v / q$ denotes the "average time valuation." The first-order conditions $\partial W / \partial q_B = 0$ and $\partial W / \partial q_L = 0$ then implicitly determine the first-best passenger quantities.[8]

These first-order conditions can be written as (asterisks for the first-best solution),

$$B_B' = vC + q^* \bar{v} C' \quad \text{and} \quad B_L' = \alpha v C + q^* \bar{v} C', \tag{4}$$

where the left-hand sides (LHSs) are the marginal passenger benefits and the right-hand sides (RHSs) are the marginal congestion costs. Observe that the marginal congestion costs depend on whether the additional passenger is of the business or leisure type. This is because an increase in the business or leisure quantity raises congestion costs for two reasons: first, it directly implies that more passengers incur delays, and the associated cost increase clearly depends on the passenger's type

[7] Here, and below, if the indices $x$ and $y$ appear in the same expression, then it is to be understood that $y \neq x$.

[8] To ensure the existence of a unique solution, welfare is assumed to be strictly concave in the business and leisure quantities.

in terms of time valuation. This effect is captured by terms $vC$ and $\alpha vC$. Second, it increases delays and congestion costs for all passengers, which is independent of the passenger's type and is captured by $q^* \bar{v} C'$.

The demands in (2) imply that passengers will internalize the congestion costs they incur themselves (i.e., $vC$ and $\alpha vC$) but won't do so for the congestion costs imposed on the other passengers (i.e., $q^* \bar{v} C'$). For this reason, term $q^* \bar{v} C'$ is called the *external part of the marginal congestion cost* (or in short, the *marginal external congestion cost* ). Using (2), first-order conditions (4) can be rewritten as

$$P_x = q^* \bar{v} C', \quad x = B, L. \tag{5}$$

This shows that passenger quantities are socially optimal if fares are the same (i.e., uniform) and are equal to the marginal external congestion cost.[9] Nonetheless, the generalized prices are discriminating at first-best optimum where $p^* = q^* \bar{v} C'$, since the business passengers' generalized prices, $\eta_B$, then become $p^* + vC$, while the leisure passengers' generalized prices, $\eta_L$, become $p^* + \alpha vC$. In other words, the *discriminating* generalized prices are required to reach the first-best solution.

### 3.2. Second-best discriminating fares

The previous analysis showed that uniform fares are necessary to achieve the first-best solution; thus, discriminating fares cannot reach the first-best solution. Suppose that prices are not uniform but third-degree discriminating (and that all markets are covered). A natural question then is how the second-best discriminating fares are related to the first-best uniform fares. To address this question, it is useful to write welfare depending on fares $p_B$ and $p_L$ (where $p_B$ and $p_L$ can be distinct). To do this, solve $p_B = P_B$ and $p_L = P_L$ for $q_B$ and $q_L$, yielding the business and leisure demands denoted, respectively, as $D_B$ and $D_L$, which depend on business and leisure fares, i.e. $D_x \equiv D_x(p_B, p_L)$.[10] Welfare as a function of business and leisure fares can then be obtained by substituting passenger quantities $q_x$ by $D_x$ in (3), which yields $W(p_B, p_L) \equiv W(D_B(p_B, p_L), D_L(p_B, p_L))$.

We already know that the welfare-optimal business and leisure fares are uniform and equal ot the marginal external congestion costs. In order to analyze second-best discriminating fares, price discrimination in the sense that $p_B > p_L$ (which will be the relevant case for us) must therefore be exogenously imposed. To do this, we make use of the "price-difference constraint." This constraint implies that business passengers are charged with a premium on top of the leisure fare that is exogenous and is determined by $\phi \geqslant 0$ ($p_B \equiv p_L + \phi$). Parameter $\phi$ can then be used to analyze uniform and discriminating fares in a unifying framework: Uniform pricing is imposed when $\phi = 0$, while $\phi > 0$ implies (strict) price discrimination. The main advantage of this unifying framework is that the comparison of fares under uniform pricing and price discrimination can be derived by a simple comparative-static analysis in $\phi$. For example, if an increase in $\phi$ is unambiguously associated with an increase of $p_x$, then we know that a change from uniform pricing to price discrimination will increase the fare in market $x$. Observe that $\phi$ determines the differential between fares in the business and leisure markets (rather than the level of fares).

The second-best discriminating fares can then be derived by analysis of the Lagrangian

$$\mathcal{L}^W(p_B, p_L) \equiv W(p_B, p_L) + vg(p_B, p_L), \tag{6}$$

where superscript $W$ indicates that welfare is the objective, $v$ is the Lagrange multiplier and the price-difference constraint is written as $g(p_B, p_L) \equiv \phi - (p_B - p_L)$. The second-best discriminating fares are determined by the first-order conditions $\mathcal{L}_x^W = 0$ (the subscript indicates the partial derivative, i.e. $\mathcal{L}_x^W \equiv \partial \mathcal{L}^W / \partial p_x$) and are denoted as $p_B^d$ and $p_L^d$, for business and leisure passengers, respectively ($d$ for discriminating). To ensure the concavity of welfare in the business and the leisure fares, assume that welfare satisfies $W_{xx} < -|W_{xy}| < 0$ for $x = B, L$ ($W_x \equiv \partial W / \partial p_x$), which leads to the following insights about the structure of the second-best discriminating fares relative to the first-best fares (the proofs of lemmas and propositions are delegated to Appendix A):

**Proposition 1.** *The second-best discriminating business fare exceeds the first-best uniform fare (which is equal to the marginal external congestion cost), while the second-best discriminating leisure fare is smaller than the first-best uniform fare, i.e.* $p_B^d > p^* > p_L^d$ *for* $\phi > 0$.

It seems intuitive that the first-best uniform fare is enclosed by second-best discriminating fares. Note that the consideration of the second-best discriminating prices imposed by a social maximizer extends the literature on third-degree price discrimination, which typically concentrates on the pricing behavior of firms.

[9] Recall that the carriers' operating costs are normalized to zero in order concentrate on congestion effects. If business passengers would receive a better service (e.g., higher quality of food and beverages, free newspapers, etc.), then marginal cost and also the first-best fares would be increased for business passengers relative to first-best fares for leisure passengers.

[10] One can check that the Jacobian of the inverse demand system (2) is negative definite, which ensures the invertibility (e.g., Vives, 1999). See Czerny and Zhang (2011, 2014) for a derivation of the comparative-static relationships between passenger quantities and fares.

## 4. Carriers' pricing behavior

This section considers the airport charge as given and analyzes carriers that compete in quantities a la Cournot under both uniform pricing and "laissez faire." The latter means that the carriers' premium charged to business passengers on top of the leisure fare is determined by carrier competition without any constraint on its size (thus, laissez faire).

### 4.1. Uniform pricing

Under uniform pricing, carriers charge the same fare, denoted as $p$ with $p_B = p_L = p$, to all passengers. The relationship between the uniform fare $p$ and aggregate quantity $q$ can be derived by solving $q = D_B(p,p) + D_L(p,p)$ for $p$, which leads to the inverse demand $p = P \equiv P(q)$. The carrier profits can now be expressed as (superscript $u$ for uniform pricing),

$$\pi_i^u = (P - \tau)q_i \tag{7}$$

for $i = 1, \ldots, n$.

The carriers' equilibrium behaviors under uniform pricing are determined implicitly by the first-order conditions, $\partial\pi_i^u/\partial q_i = 0$, which can, using symmetry, be written as

$$P = \tau - \frac{q^u P'}{n} \tag{8}$$

with $P' < 0$ (downward-sloping demand).[11] This suggests that carriers internalize their self-imposed congestion costs, which depend on market shares $(1/n)$. Further, subsidies may be required to achieve the first-best quantities, which depend not only on market shares but also on demand elasticities (e.g., Brueckner, 2002; Pels and Verhoef, 2004; Zhang and Zhang, 2006).[12] The less obvious is the dependency between the internalization and time valuations. Here, Czerny and Zhang (2011) have found that the existence of passengers with distinct time valuations can reduce the carriers' incentives to internalize self-imposed congestion.

To provide an intuitive explanation, let $\hat{v} \equiv (q_B' + \alpha q_L')v/q'$ denote the arithmetic mean of the incremental passengers' time valuations, with $q_x' \equiv dq_x/d\eta_x = 1/B_x'' < 0$ and $q' \equiv q_B' + q_L'$. With $\hat{v}$, which is to be referred to as the "marginal time valuation," first-order condition (8) can be rewritten as

$$P = \tau + \frac{1}{n}\left(q^u \hat{v} C' - \frac{q^u}{q'}\right). \tag{9}$$

Thus the carriers evaluate marginal external congestion costs at the marginal time valuation rather than the average time valuation, and if the marginal time valuation is small relative to the average time valuation, this reduces the carriers' incentive to internalize congestion costs.[13] In this situation, a high airport charge relative to the marginal external congestion cost may be needed in order to induce the first-best aggregate passenger quantity. An important consequence is that the welfare losses might be low or even be non-existent when policy makers would abstract away from market shares and simply implement the atomistic airport charge

### 4.2. Laissez faire

The laissez faire regime differs from the uniform pricing in that the carriers do not choose the sum of individual business and leisure quantities, $q_i$, but choose individual business and leisure quantities $q_{Bi}$ and $q_{Li}$, respectively, as if markets were separated. Markets can be considered as separated under laissez faire because business and leisure prices do not have to be the same and there is no constraint on the difference between business and leisure fares, while they are not entirely separated because of the demand interdependencies due to congestion.

Specifically, under laissez faire carriers choose business and leisure quantities to maximize profit (superscript *lf* indicates the laissez-faire scenario),

$$\pi_i^{lf} \equiv q_{Bi}P_B + q_{Li}P_L - \tau q_i \tag{10}$$

for $i = 1, \ldots, n$. In this situation, carrier behavior is determined by the first-order conditions, $\partial\pi_i^{lf}/\partial q_{Bi} = 0$ and $\partial\pi_i^{lf}/\partial q_{Li} = 0$ (assuming the second-order conditions and other regularity conditions hold). Using symmetry these first-order conditions can be expressed as,

$$P_x = \tau + \frac{1}{n}\left(q^{lf}\bar{v}C' - \frac{q_x}{q_x'}\right). \tag{11}$$

To ensure that the carriers charge business passengers with a high fare relative to leisure passengers (i.e., $P_B > P_L$), we assume that the business demand is less price elastic than the leisure demand in the sense that the following *elasticity condition*,

[11] Czerny and Zhang (2011, 2014) show that $P' = (B_B''B_L'' - (B_L'' + B_B''\alpha)vC')/(B_B'' + B_L'')$.

[12] While the elasticity of demand with respect to the full price is generally finite in reality, researchers (e.g., Brueckner and van Dender, 2008; Brueckner, 2009; Basso and Zhang, 2010) have considered models with perfectly elastic demands so as to concentrate their analysis on pure congestion pricing.

[13] Eq. (9) extends the monopoly case considered by Czerny and Zhang (2014) to the oligopoly case with $n$ identical carriers.

$$-q_B'/q_B < -q_L'/q_L, \tag{12}$$

holds in equilibrium in the remainder of the paper.[14]

The first-order condition (11) shows that the laissez-faire behavior is determined by the marginal external congestion costs evaluated at the average time valuation. To understand the carriers' incentives to internalize self-imposed congestion under laissez-faire relative to uniform pricing, note that the elasticity condition (12) implies that the incremental passenger comprises a higher share of leisure passengers than the inframarginal passenger, which implies $-q_L/q_L' < -q/q'$.[15] Thus, the carriers' incentive for self-internalization is, on the one hand, increased by laissez faire because carriers evaluate marginal external costs at the average congestion costs, while it is reduced because the choice of leisure quantities is determined by the leisure demand elasticity (which is small relative to $-q/q'$ in the sense that $-q_L/q_L' < -q/q'$).

The problem here is that, with laissez faire pricing, the first-order conditions cannot simply be evaluated at welfare-optimal quantities to derive a closed-form expression for the socially optimal airport charge. This is because the welfare-optimal fares are uniform and given by the marginal external congestion costs, which can never be true under strict carrier price discrimination implied by laissez faire.

## 5. Airport charge

This part of the paper considers a two-stage game, where the airport acts as a social maximizer and chooses the airport charge $\tau$ in order to maximize welfare in the first stage anticipating the carriers' equilibrium behavior in the second stage.[16] The set-up of the second stage is special in the sense that carriers are assumed to maximize individual profits by the choice of business and leisure quantities subject to the price-difference constraint. The second-stage subgame thus involves the uniform pricing and laissez faire scenarios as special cases. Importantly, this part brings the social maximizer's behavior and the carriers' behaviors together: While the social maximizer cannot directly choose fares, she chooses the airport charge to internalize congestion externality and to correct carrier market power, which then indirectly determines fares. Furthermore, since fares are discriminating under laissez faire, price discrimination can be exogenous to the social maximizer.

### 5.1. Second-stage subgame

In the second stage, the carriers choose business and leisure quantities to maximize profit; that is, carriers compete in quantities a la Cournot. The subgame-perfect equilibrium of the game will be examined in order to derive the effect of the airport charge on equilibrium behavior in terms of quantities and fares. We intentionally abstain from a comparison of prices, quantities and welfares under uniform pricing and laissez faire, which is discussed in a companion paper (Czerny and Zhang, 2014).[17]

Carriers choose quantities $q_{Bi}$ and $q_{Li}$ to maximize their profits $\pi^i$, given in (10), under the (strict) price-difference constraint $g(P_B, P_L) = 0$ with

$$g(P_B, P_L) \equiv \phi - (P_B - P_L) \tag{13}$$

and $\phi \geqslant 0$.[18] The associated Lagrangians can be written as

$$\mathcal{L}^i \equiv \pi^i + \lambda^i g, \tag{14}$$

where $\lambda^i$ are Lagrange multipliers. The Cournot–Nash quantities are determined by the first-order conditions $\mathcal{L}^i_{xi} = 0$ (where subscripts indicate partial derivatives,i.e. $\partial\mathcal{L}^i/\partial q_{xi} \equiv \mathcal{L}^i_{xi}$).[19] Part of the comparative-static results of (equilibrium) passenger quantities with respect to the airport charge are summarized in Lemma 1:

**Lemma 1.** *An increase in the airport charge has the following effects: (i) it reduces both the leisure quantity and the aggregate passenger quantity when the business time valuation is high relative to the leisure time valuation (i.e., when $\alpha < 1$); and (ii) it increases both the business and passenger fare.*

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[14] Note that the demand elasticities of business and leisure passengers with respect to generalized prices (in absolute values) can be written as $-\eta_x q_x'/q_x$. Thus the business market exhibits not only a higher time valuation than the leisure market, but also a less price-elastic demand. Lazarev (2013) finds that the business demand in the airline industry is significantly less price elastic than the leisure demand. Furthermore, $p_B^{lf} \geqslant p_L^{lf}$ may be justified by such practices as advanced-purchase rebates for leisure passengers. To abstract away from self selection, it may be assumed that the cost of early booking is prohibitive for business passengers.

[15] Substitute $q_B' + q_L'$ for $q'$ and $q_B + q_L$ for $q$, in order to rewrite the inequality $-q_L'/q_L > -q'/q$ as $-q_L'/q_L > -(q_B' + q_L')/(q_B + q_L)$. Rearranging yields $(q_L q_B' - q_L' q_B)/q_L(q_B + q_L) > 0$, where the LHS is positive by the elasticity condition.

[16] While in many real cases public subsidies are not available for infrastructure providers because of limited public funds, negative values of $\tau$ may still be considered in our context so as to simplify some of the analysis.

[17] Czerny and Zhang's (2014) main result is the identification of the time-valuation effect of price discrimination, which can work in the opposite direction as the well-known output effect on welfare. This time-valuation effect clearly explains why discriminating prices can improve welfare even when this is associated with a reduction in the aggregate passenger quantity.

[18] $\phi = 0$ implies $P_B = P_L$, which means that uniform pricing is considered, while a non-binding price-difference constraint replicates laissez faire. In other words, the price-difference constraint imposes an upper bound on $P_B$, and $\phi = 0$ forces $P_B$ to be no larger than $P_L$.

[19] Second-order and stability conditions are discussed in Appendix A (proof of Lemma 1 below).

While the effects on the leisure and aggregate quantities are intuitive and definite, the impact on the business quantity is ambiguous in general. These comparative-static results are independent of whether laissez faire or uniform pricing is considered.[20] They do depend on the difference in time valuations, however. More specifically, a nonnegative relationship between the business quantities and airport charge can occur although the business fare increases in $\tau$ (part (ii) of Lemma 1); but, this can only be true when business passengers exhibit a strictly greater value of time than leisure passengers (i.e., when $\alpha < 1$). This is because an increase in $\tau$ always reduces the aggregate passenger quantity, which reduces congestion and thus increases business demand for given fares. A positive relationship between business quantities and $\tau$ can, therefore, exist when business passengers exhibit a sufficiently high time value relative to leisure passengers so that the congestion reduction leads to a strong increase in business demand. To be precise, the relationships in (34b) (in Appendix B) imply that this occurs when $(1-\alpha)v > -B''_L/C'$. Note that the RHS is a positive term. Finally, the implications of Lemma 1 for the relationship between the airport charge and the generalized prices, $\eta_x$, are straightforward.

### 5.2. First-stage behavior

The airport (social maximizer) does not directly control quantities or fares; fares are determined by carriers. Rather, the airport controls the airport charge, which is taken as given by carriers in their second-stage Cournot rivalry (examined in the above section). In what follows, the welfare-optimal airport charge in the two-stage game is identified by backward induction.

In stage one, the carriers' equilibrium behavior in stage two is anticipated correctly by the airport and $\tau$ is chosen to maximize welfare. The welfare-optimal choice of $\tau$ under airline price discrimination ($\phi$ is considered as given), denoted as $\tau^d$, is determined by the first-order condition

$$\frac{dW}{d\tau} = W_B \frac{dq_B}{d\tau} + W_L \frac{dq_L}{d\tau} = 0, \tag{15}$$

which implies:

**Lemma 2.** *The welfare-optimal airport charge leads to the second-best discriminating business and leisure fares $p_B^d$ and $p_L^d$.*

An increase in the airport charge increases the leisure and the business fare by Lemma 1, while the difference between fares is exogenous; thus, the airport charge can be chosen to implement the second-best discriminating fares. This result can be used to derive the welfare-optimal airport charge as follows. Letting $q_B^d$ and $q_L^d$ denote the second-best behavior in terms of business and leisure quantities (i.e., $q_B^d \equiv D_B(p_B^d, p_L^d)$ and $q_L^d \equiv D_L(p_B^d, p_L^d)$) respectively, it holds that:

**Proposition 2.** *For $\phi \geqslant 0$, the second-best airport charge that maximizes welfare conditional on carrier price discrimination imposed by the price-difference constraint can be written as*

$$\tau^d = \left(1 - \frac{1}{n}\right) q^d \bar{v} C' + \frac{q^d B''_B B''_L + (q_B^d B''_B - q_L^d B''_L)(1-\alpha) v C'}{n(B''_B + B''_L)}. \tag{16}$$

The optimal airport charge (16) is inversely related to market shares $(1/n)$. For instance, when $n \to \infty$, the optimal airport charge is equal to the external part of the marginal congestion costs, $q^d \bar{v} C'$. Moreover, since $n \to \infty$ implies an atomistic market structure, fares in the business market and fares in the leisure market are determined by the airport charge and thus $p_B^d = p_L^d = q^d \bar{v} C' = q^* \bar{v} C'$, yielding the first-best result. Again, two reasons can be identified for the relationship between the optimal airport charge and market shares: first, carriers entirely or partly internalize marginal congestion costs if these are self-imposed. Second, there is a "market power" effect: When elasticities are finite so that equilibrium fares increase in market concentration, the airport charge should be small or even negative so as to induce low fares downstream, thereby correcting for carrier market power.

As indicated earlier, the existing literature on airport congestion pricing mainly concentrates on a single type of passengers in the sense that passengers are assumed to have the same time valuations. To elaborate on the effect of passenger groups, suppose that $\alpha = 1$. In this instance, the second term on the RHS of (16) is clear-cut in sign and is negative, which implies the existence of an upper limit for the optimal airport charge determined by $(1 - 1/n) q^d \bar{v} C'$. By contrast, if $\alpha < 1$, then $\tau^d > (1 - 1/n) q^d \bar{v} C'$ when

$$(1-\alpha) v > -\frac{(q_B^d + q_L^d) B''_B B''_L}{(q_B^d B''_B - q_L^d B''_L) C'}, \tag{17}$$

where the RHS is strictly positive, since $q_B^d B''_B - q_L^d B''_L = q_B^d / q'_B - q_L^d / q'_L < 0$ holds by the elasticity condition.

Thus, with the difference in time valuation, $(1 - 1/n) q^d \bar{v} C'$ cannot be considered as an upper limit for the optimal airport charge anymore. Since this result holds for $\phi \geqslant 0$, it is true under uniform pricing as well as under price discrimination. This shows that the result obtained by Czerny and Zhang (2011), who concentrated on uniform fares and showed that an increase in $\tau$ can increase welfare by protecting high-time-value passengers from excessive congestion caused by low-time-value passengers, is also true when airlines price discriminate.

[20] Similar setting has been analyzed by Czerny and Zhang (2011) for uniform pricing.

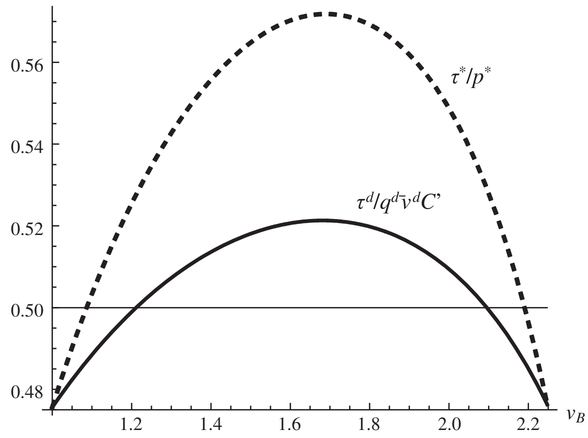


**Fig. 1.** Ratios of the first- (dashed line) and second-best (solid line) optimal airport charges and marginal external congestion costs. Parameters: $a = 8/5,\ b_B = 2,\ b_L = 1/20,\ n = 2,\ v_L = 1$.

Summarizing the above discussion yields:

**Proposition 3.** *When the business time valuation exceeds the leisure time valuation (i.e., $\alpha < 1$), the optimal airport charge can be higher than what would prevail when passengers were treated as of a single type (in the sense that time valuations are taken to be the same for all passengers). Furthermore, this result holds for $\phi \geqslant 0$ and therefore holds under uniform pricing and laissez faire.*

For an intuition, note that the incentives to internalize self-imposed congestion under uniform pricing and laissez faire are closely related to each other. To see this, recall that carriers are concerned with marginal time valuations $\hat{v}$ under uniform pricing, while they are concerned with average time valuations $\bar{v}$ under laissez faire. The incentives to internalize self-imposed congestion are therefore reduced under uniform pricing when the marginal time valuation is low relative to the average time valuation (i.e., $\hat{v} < \bar{v}$). On the other hand, they are reduced under laissez faire relative to uniform pricing when the proportion of leisure passengers is higher for incremental than for inframarginal passengers (i.e., $-q_L/q_L' < -q/q'$). Note that these conditions are directly related: If the incentives to internalize self-imposed congestion are low under laissez faire, i.e. $-q_L/q_L' < -q/q'$, this implies $q_L'/q' > q_L/q$ and $\hat{v} < \bar{v}$ when time valuations are distinct (Czerny and Zhang, 2014), which means that the incentives to internalize self-imposed congestion are also reduced when prices are uniform. Furthermore, $-q_L/q_L' < -q/q'$ and, thus, $\hat{v} < \bar{v}$ is implied by the elasticity condition when time valuations are distinct. Thus, by imposing a high leisure demand elasticity relative to the business elasticity, the elasticity condition reduces the carriers' incentives for self-internalization under both uniform pricing and laissez faire.

### 5.3. Examples

Specific functional forms are used to further analyze the relationship between the welfare-optimal airport charge and price discrimination. For instance, quadratic benefits

$$B_B = aq_B - \frac{b_B}{2}q_B^2 \quad \text{and} \quad B_L = q_L - \frac{b_L}{2}q_L^2 \tag{18}$$

with $a, b_B, b_L > 0$ and with $C = q$ imply a rather extensive output for $\tau^d$ (which is omitted here), while this difficult expression leads to the compact and easy to interpret derivative

$$\frac{d\tau^d}{d\phi} = -\frac{(1-\alpha)v}{b_B + b_L} \leqslant 0. \tag{19}$$

This derivative shows that the first-best airport charge under uniform pricing exceeds the second-best airport charge under price discrimination when demands are linear. Note that laissez faire reduces the aggregate passenger quantity relative to uniform pricing (Czerny and Zhang, 2014), which reduces congestion under laissez faire and thus the need to internalize congestion externalities by an increase in the airport charge. The effect of the pricing regime on the aggregate quantity and thus the welfare-optimal airport charge depends however strongly on the curvature of demands.[21] Specifically, Appendix C provides an example with a concave business demand, where the welfare-optimal airport charge under laissez faire is higher than under uniform pricing.

[21] How the demand curvatures affect the aggregate quantity under laissez faire relative to uniform pricing has been analyzed by Robinson (1933), Schmalensee (1981), Holmes (1989) and Stole (2007) for independent demands. More recently, by assuming independent demands as well, Aguirre et al. (2010) derived conditions for the curvature of demand functions that are informative with respect to the welfare effect of price discrimination. Cowan (2013) analyzed the welfare and output effects of price discrimination for the special case of parallel inverse demand functions.

To illustrate how closely related the incentives to internalize self-imposed congestion are under uniform pricing and laissez faire, the following example describes the relationship between marginal external congestion costs and the welfare-optimal airport charge. Recall that the welfare-optimal airport charge (16) implies $\tau^*/p^* \leqslant 1/2$ when $\alpha = 1$. By contrast, Fig. 1 shows that parameter constellations exist, where $\tau^*/p^* > 1/2$ or, respectively, $\tau^d/(q^d\bar{v}C') > 1/2$ when $\alpha < 1$. (parameters are $a = 8/5, b_B = 2, b_L = 1/20, n = 2$ and $v_L = 1$). This demonstrates that it can be useful to increase the airport charge to a level that exceeds the external marginal congestion costs in order to correct the incentives for self-internalization, which can be reduced under *both* uniform pricing and laissez faire when passenger groups with distinct time valuations exist. Furthermore, the shapes of the dashed and solid lines in Fig. 1 are similar, which illustrates that the carriers' incentives for self-internalization under both uniform pricing and laissez faire are closely related to each other.

## 6. Concluding remarks

This paper has investigated the second-best carrier behavior that maximizes welfare conditional on carrier price discrimination. It has further derived the second-best optimal airport charge that implements the second-best carrier behavior and compared the first- and second-best carrier and airport behaviors. To do this, we extended the model developed by Czerny and Zhang (2014) in order to analyze a two-stage game where the airport charge is chosen to maximize welfare in the first stage, while carriers compete in quantities subject to a price-difference constraint in the second stage.

We found that the second-best discriminating business fare always exceeds the first-best uniform fare, while the second-best discriminating leisure fare is always lower than the first-best uniform fare. Furthermore, the second-best behavior can be implemented by the right choice of the airport charge. Surprisingly, the basic structure of the optimal airport charge is independent of whether carriers engage in price discrimination or just charge uniform fares. This is true in the sense that with or without price discrimination, the welfare-optimal airport charge can be higher than what would be expected when all passengers were treated as having the same time valuation (which has so far been the practice in the existing literature). This robustness arises from the fact that the carriers' incentives to internalize self-imposed congestion depend on demand elasticities and the difference between the business and the leisure passengers' time valuations, which is true under uniform pricing and laissez faire. This shows that the relationship between time valuations and airport charges found by Czerny and Zhang (2011) is robust with respect to carriers' pricing behavior. The analysis thus provides some support to the finding that the welfare losses may be low if policy makers would simply implement the atomistic airport congestion charge (even when carriers have market power).

A natural avenue for future research is to consider airport price discrimination with respect to passenger types. The challenges here are to identify the optimal discriminating airport charges and to design a charging scheme that may be implementable in practice (recall that airline price discrimination is often based on the timing of ticket purchases, which may not be a practical approach for airport price discrimination).

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## Appendix A. Derivatives

The second and cross derivatives of the carriers' Lagrangians in the second subgame can be written as:

$$\mathcal{L}^i_{LiLi} = 2(B''_L - \alpha vC') + q_{Li}B'''_L - \Big(q_{Bi} + \alpha q_{Li} - \lambda^i(B'''_L + (1-\alpha))\Big)vC'', \tag{20}$$

$$\mathcal{L}^i_{LiLj} = B''_L - \alpha vC' + q_{Li}B'''_L - \Big(q_{Bi} + \alpha q_{Li} - \lambda^i(B'''_L + (1-\alpha))\Big)vC'', \tag{21}$$

$$\mathcal{L}^i_{BiLi} = -(1+\alpha)vC' - \Big(q_{Bi} + \alpha q_{Li} - \lambda^i(1-\alpha)\Big)vC'', \tag{22}$$

$$\mathcal{L}^i_{BiLj} = -vC' - \Big(q_{Bi} + \alpha q_{Li} - \lambda^i(1-\alpha)\Big)vC'', \tag{23}$$

$$\mathcal{L}^i_{BiBi} = 2(B''_B - vC') + q_{Bi}B'''_B - \Big(q_{Bi} + \alpha q_{Li} + \lambda^i(B'''_B - (1-\alpha))\Big)vC'', \tag{24}$$

$$\mathcal{L}^i_{BiBj} = B''_B - vC' + q_{Bi}B'''_B - \Big(q_{Bi} + \alpha q_{Li} + \lambda^i(B'''_B - (1-\alpha))\Big)vC'', \tag{25}$$

$$\mathcal{L}^i_{LiBj} = -\Big(\alpha vC' + \Big(q_{Bi} + \alpha q_{Li} - \lambda^i(1-\alpha)\Big)vC''\Big). \tag{26}$$

## Appendix B. Proofs

**Proof of Proposition 1.** Given that the second-best discriminating fares are determined by the first-order conditions $\mathcal{L}_x^W = 0$, we show that $dp_B^d/d\phi > 0$, while $dp_L^d/d\phi < 0$. Denote the determinant of the bordered Hessian of $\mathcal{L}^W$ with respect to business and leisure fares as

$$\Upsilon \equiv \det\begin{pmatrix} \mathcal{L}_{BB}^W & \mathcal{L}_{BL}^W & \frac{\partial g}{\partial p_B} \\ \mathcal{L}_{BL}^W & \mathcal{L}_{LL}^W & \frac{\partial g}{\partial p_L} \\ \frac{\partial g}{\partial p_B} & \frac{\partial g}{\partial p_L} & 0 \end{pmatrix} = \det\begin{pmatrix} W_{BB} & W_{BL} & -1 \\ W_{BL} & W_{LL} & 1 \\ -1 & 1 & 0 \end{pmatrix} \tag{27a}$$

$$= -(W_{LL} + 2W_{BL} + W_{BB}), \tag{27b}$$

where the RHS is positive, since $W_{xx} < -|W_{xy}| < 0$ for $x = B, L$ by assumption, and ensures the existence of a solution.[22] Cramer's rule can be used to derive

$$\frac{dp_B^d}{d\phi} = \frac{1}{\Upsilon}\det\begin{bmatrix} 0 & W_{BL} & -1 \\ 0 & W_{LL} & 1 \\ -1 & 1 & 0 \end{bmatrix} = -\frac{1}{\Upsilon}(W_{BL} + W_{LL}) > 0 \tag{28}$$

and

$$\frac{dp_L^d}{d\phi} = \frac{1}{\Upsilon}\det\begin{bmatrix} W_{BB} & 0 & -1 \\ W_{BL} & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \frac{1}{\Upsilon}(W_{BB} + W_{BL}) < 0. \tag{29}$$

**Proof of Lemma 1.** The Cournot–Nash quantities are determined by the first-order conditions $\mathcal{L}_{xi}^i = 0$. Using symmetry, these conditions can be written as

$$(B_x' - \upsilon_x C - \tau) + B_x'' q_{xi} - \frac{1}{n} q \bar{\upsilon} C' + \lambda^i g_x = 0 \tag{30}$$

with

$$g_B \equiv -B_B'' + (1-\alpha)\upsilon C' \quad \text{and} \quad g_L \equiv B_L'' + (1-\alpha)\upsilon C'. \tag{31}$$

The first equation in (31) is positive in sign, while the second equation is ambiguous in sign when $\alpha < 1$. To ensure the existence of solutions for each carrier's choice of passenger quantities, assume that the second-order conditions are satisfied, that is, the bordered Hessians satisfy

$$\det\begin{bmatrix} \mathcal{L}_{BiBi}^i & \mathcal{L}_{BiLi}^i & g_B \\ \mathcal{L}_{BiLi}^i & \mathcal{L}_{LiLi}^i & g_L \\ g_B & g_L & 0 \end{bmatrix} > 0, \tag{32}$$

where the second and cross derivatives of the Lagrangians $\mathcal{L}^i$ with respect to individual business and leisure quantities are provided in Appendix A. Equilibrium outcomes are characterized by simultaneously solving the first-order conditions in (30) for $i = 1, \ldots, n$ and by imposing symmetry and the assumption that,

$$\Xi_i \equiv \det\begin{bmatrix} \mathcal{L}_{BiBi}^i + (n-1)\mathcal{L}_{BiBj}^i & \mathcal{L}_{BiLi}^i + (n-1)\mathcal{L}_{BiLj}^i & g_B \\ \mathcal{L}_{LiBi}^i + (n-1)\mathcal{L}_{LiBj}^i & \mathcal{L}_{LiLi}^i + (n-1)\mathcal{L}_{LiLj}^i & g_L \\ n g_B & n g_L & 0 \end{bmatrix} > 0 \tag{33}$$

for $i = 1, \ldots, n$ at equilibrium.[23]

To establish part (i), totally differentiate the first-order conditions (30) with respect to $\tau$ and use symmetry as well as Cramer's rule, which yields

[22] To ensure that a solution exists when a maximization problem involves $k$ variables and $l$ constraints ($l < k$), the bordered principal minor of the Hessian determinant $|H|$, denoted as $|H_{l+j}|$, must have the sign $(-1)^{l+j}$ for $j = 1, \ldots, k-l$ (e.g., Gravelle and Reese, 2004). In our case $k = 2$ and $l = 1$ due to the symmetry, and $|H_2|$ must be positive in sign, since $(-1)^2 = 1$.

[23] Condition (33) reduces to condition (32) when $n = 1$. For $n > 1$ and non-binding constraint $g = 0$ it is a sufficient stability condition for the two-market Cournot equilibrium (Zhang and Zhang, 1996), and so it may be considered as a generalized stability condition with the constraint (noting $g_{xi} = g_{xj}$). Condition (33) is satisfied when $B_B''' \leqslant 0, B_L''' \leqslant 0$ and $\upsilon_B = \upsilon_L$, and it also holds with the specific functional forms used in this paper.

$$\frac{dq_B}{d\tau} = \frac{n}{\Xi_i}\det\begin{bmatrix} 1 & \left(\mathcal{L}^i_{BiLi} + (n-1)\mathcal{L}^i_{BiLj}\right) & g_B \\ 1 & \left(\mathcal{L}^i_{LiLi} + (n-1)\mathcal{L}^i_{LiLj}\right) & g_L \\ 0 & ng_L & 0 \end{bmatrix} \tag{34a}$$

$$= -\frac{n^2}{\Xi_i}\left(B''_B + B''_L\right)\left(B''_L + (1-\alpha)\upsilon C'\right), \tag{34b}$$

where the RHS is ambiguous in sign when $\alpha < 1$, while the RHSs of both

$$\frac{dq_L}{d\tau} = -\frac{n^2}{\Xi_i}\left(B''_B + B''_L\right)\left(B''_B - (1-\alpha)\upsilon C'\right) \tag{35}$$

and

$$\frac{d(q_B + q_L)}{d\tau} = -\frac{n^2}{\Xi_i}\left(B''_B + B''_L\right)^2 \tag{36}$$

are clear-cut and negative in sign.

To derive the relationships between fares and the airport charge, recall that $P_B = P_L + \phi$ and, thus, $dP_B/d\tau = dP_L/d\tau$ with $dP_x/d\tau = B''_x dq_x/d\tau - \upsilon_x C' dq/d\tau$. More specifically, the relationships in (34b), (35) and (36) together with inverse demands in (2) yield

$$\frac{dP_x}{d\tau} = -\frac{n^2}{\Xi_i}\left(B''_B + B''_L\right)\left(B''_B B''_L - \left(B''_B + \alpha B''_L\right)\upsilon C'\right) \tag{37}$$

for $x = B, L$, where the RHS is clear-cut and positive in sign, which establishes part (ii).

**Proof of Lemma 2.** To show that the second-best fares can be implemented by the right choice of the airport charge, we show that the second-best carrier behavior in terms of business and leisure quantities can be implemented by the right choice of the airport charge.[24] The welfare-optimal passenger quantities when airlines price discriminate are obtained by maximizing the Lagrangian $\mathcal{L}^q \equiv W(q_B, q_L) + \mu g(P_B, P_L)$ with respect to $q_B$ and $q_L$, where $\mu$ is the Lagrange multiplier. The corresponding first-order conditions can be written as

$$\mathcal{L}^q_B = W_B + \mu g_B \tag{38a}$$

$$= p^d_B - q^d \bar{\upsilon} C' + \mu\left(-B''_B + (1-\alpha)\upsilon C'\right) = 0 \tag{38b}$$

and

$$\mathcal{L}^q_L = W_L + \mu g_L \tag{39a}$$

$$= p^d_L - q^d \bar{\upsilon} C' + \mu\left(B''_L + (1-\alpha)\upsilon C'\right) = 0. \tag{39b}$$

To show that $\tau^d$ leads to $p^d_B$ and $p^d_L$, multiply the first-order conditions in (38a) and (39a) by $dq_B/d\tau$ or $dq_L/d\tau$, respectively. Summing the first-order conditions then yields

$$W_B\frac{dq_B}{d\tau} + W_L\frac{dq_L}{d\tau} + \mu\left(g_B\frac{dq_B}{d\tau} + g_L\frac{dq_L}{d\tau}\right) = 0. \tag{40}$$

Note that

$$g_B\frac{dq_B}{d\tau} + g_L\frac{dq_L}{d\tau} \tag{41a}$$

$$\iff \left(-B''_B + (1-\alpha)\upsilon C'\right)\left(B''_L + (1-\alpha)\upsilon C'\right) + \left(B''_L + (1-\alpha)\upsilon C'\right)\left(B''_B - (1-\alpha)\upsilon C'\right) \tag{41b}$$

by the relationships in (34b) and (35). Simplifying shows that this expression is equal to zero, which means that $W_B dq_B/d\tau + W_L dq_L/d\tau = 0$ in optimum.

**Proof of Proposition 2.** The first-order condition in (15) can be rewritten as

$$\frac{dW}{d\tau} = P^d_B\frac{\partial q^d_B}{\partial\tau} + P^d_L\frac{\partial q^d_L}{\partial\tau} - q^d\bar{\upsilon}C'\frac{\partial q^d}{\partial\tau} = 0, \tag{42}$$

[24] Since demands are invertible, this is a valid approach.

which is equivalent to

$$p_B^d \frac{\partial q_B}{\partial \tau}\frac{\partial \tau}{\partial q^d} + p_L^d \frac{\partial q_L}{\partial \tau}\frac{\partial \tau}{\partial q^d} = q^d \bar{\upsilon} C'. \tag{43}$$

It is useful to denote

$$\alpha_x = \frac{\partial q_x^d/\partial \tau}{\partial q^d/\partial \tau} \tag{44}$$

with

$$\alpha_B = \frac{B_L'' + (1-\alpha)\upsilon C'}{B_B'' + B_L''} \quad \text{and} \quad \alpha_L = \frac{B_L'' - (1-\alpha)\upsilon C'}{B_B'' + B_L''}, \tag{45}$$

where $\alpha_B$ is ambiguous in sign, $\alpha_L$ is positive in sign and $\alpha_B + \alpha_L = 1$. Eq. (43) can now be rewritten as

$$\alpha_B p_B^d + \alpha_L p_L^d = q^d \bar{\upsilon}\, C'. \tag{46}$$

Furthermore, rearranging and multiplying the derivative in (30) by $\alpha_x$ leads to

$$p_B^d \alpha_B = \tau^d \alpha_B - \left(q_{Bi}^d \frac{\partial P_B}{\partial q_B} + q_{Li}^d \frac{\partial P_L}{\partial q_B}\right)\alpha_B + \lambda_i g_B \alpha_B \tag{47}$$

and

$$p_L^d \alpha_L = \tau^d \alpha_L - \left(q_{Bi}^d \frac{\partial P_B}{\partial q_L} + q_{Li}^d \frac{\partial P_L}{\partial q_L}\right)\alpha_L + \lambda_i g_L \alpha_L. \tag{48}$$

Summing up gives

$$\alpha_B p_B^d + \alpha_L p_L^d = \tau^d - \left(q_{Bi}^d \frac{\partial P_B}{\partial q_B} + q_{Li}^d \frac{\partial P_L}{\partial q_B}\right)\alpha_B - \left(q_{Li}^d \frac{\partial P_B}{\partial q_L} + q_{Li}^d \frac{\partial P_L}{\partial q_L}\right)\alpha_L + \lambda_i (g_B \alpha_B + g_L \alpha_L). \tag{49}$$

Note that

$$\alpha_B g_B = \frac{B_B'' B_L'' + (B_L'')^2 + (B_B'' + B_L'')(1-\alpha)\upsilon C'}{(B_B'' + B_L'')^2}\left(-B_B'' + (1-\alpha)\upsilon C'\right) \tag{50}$$

and

$$\alpha_L g_L = \frac{B_B'' B_L'' + (B_B'')^2 - (B_B'' + B_L'')(1-\alpha)\upsilon C'}{(B_B'' + B_L'')^2}\left(B_L'' + (1-\alpha)\upsilon C'\right), \tag{51}$$

which after some manipulation can be shown to imply $\alpha_B g_B + \alpha_L g_L = 0$. Deducting (46) from (49) and rearranging yields

$$\tau^d = q^d \bar{\upsilon} C' + \left(q_{Bi}^d \frac{\partial P_B}{\partial q_B} + q_{Li}^d \frac{\partial P_L}{\partial q_{Bi}}\right)\alpha_B + \left(q_{Bi}^d \frac{\partial P_B}{\partial q_L} + q_{Li}^d \frac{\partial P_L}{\partial q_L}\right)\alpha_L \tag{52a}$$

$$= \left(1 - \frac{1}{n}\right) q^d \bar{\upsilon} C' + \frac{q_B^d \left(B_L'' + (1-\alpha)\upsilon C'\right) B_B'' + q_L^d \left(B_B'' - (1-\alpha)\upsilon C'\right) B_L''}{n\left(B_B'' + B_L''\right)}, \tag{52b}$$

which, finally, leads to the second-best airport charge (16) for $\phi \geqslant 0$.

## Appendix C. Concave business demand

The inequality (19) shows that price discrimination reduces the welfare-optimal airport charge when demands are linear, and the intuition is related to the fact that the aggregate quantity is reduced by laissez faire when business and leisure demands are linear. Robinson (1933) showed that price discrimination increases the aggregate quantity when the demand in the strong market (business market in our framework) is concave, while the demand in the weak market (leisure market

**Table 1**
Outcomes at the subgame-perfect equilibrium when business demand is concave. Parameters: $\upsilon_B = 18/20,\ \upsilon_L = 17/20,\ n = 3$.

| | $\tau$ | $q_B$ | $q_L$ | $q$ | $p_B$ | $p_L$ | $\eta_B$ | $\eta_L$ |
|---|---|---|---|---|---|---|---|---|
| Uniform pricing | 0.249 | 0.284 | 0.342 | 0.626 | 0.523 | 0.523 | 1. 086 | 1.055 |
| Laissez faire | 0.252 | 0.236 | 0.365 | 0.601 | 0.606 | 0.446 | 1. 147 | 0.957 |

in our framework) is convex. The following example illustrates that the welfare-optimal airport charge can be increased by laissez faire relative to uniform pricing when the business demand is strictly concave. Suppose the benefits in the business market are given by

$$B_B = \frac{7}{2} q_B - \frac{1}{3} q_B^3, \tag{53}$$

(which lead to a concave demand in the business market) and if $v_B = 18/20$, $v_L = 17/20$ and $n = 3$, a move from uniform pricing towards price discrimination slightly increases the airport charge $\tau^d$ from 0.249 to 0.252 (see Table 1):

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