Transportation Research Part E 77 (2015) 45–60

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

Airport partial and full privatization in a multi-airport region: Focus on pricing and capacity

Mohamadhossein Noruzoliaee [a], Bo Zou [a,*], Anming Zhang [b,c]

[a] Department of Civil and Materials Engineering, University of Illinois at Chicago, USA
[b] Sauder School of Business, University of British Columbia, Canada
[c] China Academy for Financial Research, Shanghai Jiao Tong University, China

ARTICLE INFO

Article history:
Received 29 September 2014
Received in revised form 11 January 2015
Accepted 14 February 2015

Keywords:
Multi-airport systems
Airport privatization
Capacity expansion
Pricing
Public–private duopoly

ABSTRACT

This paper studies the capacity and pricing choice of two congestible airports in a multi-airport metropolitan region, under transition from a pure public, centralized airport system to partial or full privatization. We develop analytical models to investigate three privatization scenarios: public–private duopoly, private–private duopoly, and private monopoly. We find that, airports follow the same capacity investment rule as prior to privatization when capacity and pricing decisions are made simultaneously. Pricing rule after privatization becomes more complicated, with additional factors having an upward effect on the privatized airport(s) and a downward effect on the remaining public airport.

1. Introduction

This paper investigates airport capacity and pricing under privatization in a multi-airport system. Facing air traffic congestion at airports, it is well recognized that airport pricing and capacity investment are two major, intertwined solutions, which are further related to airport ownership forms (Basso and Zhang, 2007a; Zhang and Czerny, 2012). The latter has garnered increasing attentions in the research community given the wave of airport privatization worldwide over the past two decades. Although a large body of literature has been produced which deals with the joint issue of capacity investment and pricing under different ownership forms for a single airport, few studies have examined this in a multi-airport setting (De Borger and Van Dender, 2006; Basso and Zhang, 2007b; Basso, 2008).

In contrast to this scarcity of multi-airport system pricing and capacity literature is the rapid development of metropolitan regions with more than one airport. In the beginning of the 2000s, multi-airport systems worldwide already catered to about one billion passengers, well above half of the global air passenger traffic at that time (De Neufville and Odoni, 2003). The surge of multi-airport system development continues, especially in developing economies (CAPA, 2014). This rapid development, together with the lack of understanding of the way multi-airport systems function, has led to expensive and embarrassing failures in operating such systems (De Neufville and Odoni, 2003).

The combination of the worldwide growth of multi-airport systems and airport privatization gives rise to the question of how privatization will shape the capacity and pricing decisions for each airport in a multi-airport region. While some existing efforts (e.g., De Borger and Van Dender, 2006; Basso and Zhang, 2007b; Basso, 2008) attempted to provide answers,

* Corresponding author.
E-mail addresses: mnoruz2@uic.edu (M. Noruzoliaee), bzou@uic.edu (B. Zou), anming.zhang@sauder.ubc.ca (A. Zhang).

http://dx.doi.org/10.1016/j.tre.2015.02.012
1366-5545/

they have all relied on making additional assumptions about key components in the airport demand and supply. Whether their findings are generalizable remains an interesting subject to be explored.

Perhaps more importantly, the presence of multiple airports (rather than a single airport) is likely to lead to a more progressive privatization process: for example, starting with privatizing just one airport in the multi-airport system instead of a once-and-for-all switch. Indeed, coexistence of public–private ownership has been seen in a number of existing multi-airport systems, such as in Stockholm, Johannesburg, and Tehran (Bonnefoy, 2007). Two notable recent examples of partially privatizing multi-airport systems are: the proposal of privatizing Chicago Midway airport while leaving intact the public ownership of Chicago O'Hare airport; and the on-going privatization of Kansai airport in Osaka, Japan, where three airports (Kansai, Itami, and Kobe) exist in the metropolitan region. Despite these real world cases, the implications of partial privatization for airport capacity and pricing choice have not been investigated in the academic literature. Examination of such a public–private duopoly case is nonetheless important, as the existing papers have shown that pure profit maximization or pure social welfare maximization may not be appropriate given the way the airport demand and revenue functions are formulated (e.g., De Borger and Van Dender, 2006; Basso and Zhang, 2007b): the former leads to high airport charges while latter leads to government subsidies which are increasingly unlikely nowadays.

The objective of this paper is therefore to scrutinize airport capacity and pricing choice of multiple-airport systems transitioning from pure public ownership to alternative privatization scenarios, including: (1) one airport becomes privatized, while the other remains public, i.e. a public–private duopoly; (2) both airports become privatized and owned by two distinct private agencies, i.e., a private–private duopoly; and (3) both airports are sold to one private company, i.e. private monopoly. This gives rise to our main contribution: that is, our analysis provides comprehensive consideration and comparison of the aforementioned ownership structures in a multi-airport region, which advances existing knowledge about the economic implications of privatizing multi-airport systems. Our second main contribution lies in the development of a general analytical framework for investigating airport capacity and pricing choice.

Our synthetic approach allows us to offer some major insights in a *more unified way across different scenarios*, summarized as follows. First, service price is set equal to marginal social cost under pure public ownership. Once privatized, the privatized airport(s) will disregard the passenger welfare component and tend to increase service price. For the remaining public airport under partial privatization, a markdown is expected due to the competition pressure. Second, somewhat amazingly, the capacity investment rule remains simple as long as pricing and capacity decisions are made simultaneously: optimal capacity sets marginal capacity cost equal to marginal delay reduction benefits. This is invariant to airport ownership forms in a two-airport region. Third, if capacity decisions are made prior to pricing decisions, then optimal capacity is set such that marginal capacity cost equals either marginal profit or social benefit, depending on whether the airport is privatized or remains public. While the exact change in airport capacity, service price, and other supply–demand characteristics after privatization depends on parameter settings, the theoretic insights obtained from our analysis offer new pricing and capacity implications for multi-airport privatization in an intuitive way, thus helping inform policy debates on privatization of multi-airport systems.

The rest of the paper is organized as follows. A review of the theoretical literature on airport pricing and capacity investment, and privatization, in particular in multi-airport systems, is offered in Section 2. Section 3 specifies the airport demand function and examines the base case of a centralized public two-airport system. In Section 4, capacity and pricing decision-making for a two-airport region is investigated under privatization. Three possible full or partial privatization scenarios (corresponding to three different airport ownership structures) are analyzed. Section 5 provides a synthetic discussion on the impacts of privatization on airport pricing and capacity choice in a two-airport region. Concluding remarks and suggestions for future research are given in Section 6.

2. Literature review

The literature on airport pricing and capacity investment can be traced back (at least) to Levine (1969) and Carlin and Park (1970). The theoretic literature has since been significantly expanded to jointly considering capacity investment, airport concession, airport regulation, and airport–airline vertical structure. Comprehensive reviews of work in this field can be found in Basso and Zhang (2007a) and Zhang and Czerny (2012). In what follows we present a brief review of recent work on airport pricing and capacity choice, and multi-airport competition and privatization, with a view of identifying research gaps that we intend to fill in the paper.

Among the long list of studies on airport capacity and pricing choice, one of the most used criteria to categorize the literature is whether the vertical structure between an airport and the airlines using the airport is considered. The traditional approach does not consider the airport–airline vertical structure, following partial equilibrium analysis in which an airport's demand is directly a function of the airport's own decisions (Basso and Zhang, 2007a). In contrast, the vertical-structure approach incorporates airlines' market power and strategic behavior, and passenger demands in a vertical structure. The vertical-structure approach recognizes that airports provide an input for the airline market and it is the equilibrium of this downstream market that determines the airports' demand. Therefore, the demand for airport services is a derived demand. It has been proven that the traditional approach is equivalent to the vertical-structure approach when the airline market is perfectly competitive (Basso and Zhang, 2008). Other than the traditional vs. vertical-structure approach, the literature can be alternatively categorized by whether airport capacity increase is considered as discrete (e.g., Oum and Zhang, 1990; Zhang and Zhang, 2003) or continuous (Zhang and Zhang, 2006; Basso, 2008), and by whether demand is treated as deterministic (as is done in most existing research) or stochastic (Xiao et al., 2013). Focusing on pricing, operations research models have been developed which offer greater flexibility in characterizing system supply and demand (e.g., Yan and Winston, 2014; Vaze and Barnhart, 2012a).

Regarding airport privatization and airport competition, as mentioned earlier most existing studies consider a single airport. We found only a few existing studies that deal with privatization in the multi-airport context (or more generally, multiple congestible facilities). De Borger and Van Dender (2006) studied the private duopolistic interaction between two facilities that supply perfect substitutes and make sequential decisions on capacities and prices. The authors found that capacity provision under duopoly is lower and congestion levels are higher than in the social optimum. Basso and Zhang (2007b) investigated competition between two private facilities by explicitly incorporating the behavior of oligopolistic downstream carriers and ultimate consumers. The authors found that the duopoly facilities have lower prices than a private monopoly, but offer lower service quality (i.e., higher level of delays) if capacity decisions are made prior to service charge. However, if capacity and price are determined simultaneously, the duopolists offer the same service quality as a monopolist. Using again a vertical structure, more recently, Mun and Teraji (2012) examined the organization of a multiple airport system where three structures (private–private duopoly, public monopoly, and private monopoly) are compared. Their focus was on allocation of international and domestic flights among multiple airports in a metropolitan area.[1]

We also note a growing stream of research on multi-airport privatization in the context of airports located in different countries and engaged in competition. Mantin (2012) adopted a game theoretic approach to investigate airport decisions on whether to keep an airport public, or privatize it. A similar approach is employed by Matsumura and Matsushima (2012), who attempted to answer the question of when national governments have incentive to privatize public airports. Airport pricing behavior was part of the authors' discussion on public–private duopoly, but airport capacity choice and the interaction between the two airports were not included in the analysis. Further efforts on airport privatization game with more complicated network structures have been made by Lin (2013), Lin and Mantin (2014) and Kawasaki (2014), with the latter two additionally considering international inter-hub and spoke networks and a domestic airline network respectively. Beyond airports, Czerny et al. (2014) investigated the privatization decision for two competing seaports in different countries. The impacts of port privatization on user fees, firm profits, and welfare were analyzed by Matsushima and Taksuchi (2013). Overall, the focus of this line of research is on airport (port) privatization decisions. Capacity choice and congestion delays are not part of the analysis.

The review of relevant literature has made clear that all previous studies rely on the use of specific functions to characterize demand and supply in the systems considered. Joint investigation of pricing and capacity decisions together with the full range of the above-mentioned multi-airport organization forms have yet been investigated; indeed, they require further insights that are more generic and independent of specific system functions. In particular, given that partial privatization could represent a "transition" phase between pure public ownership and full privatization, understanding the implications for airport pricing and capacity choice of co-existence of public and private airports is important but is absent in the literature. This research intends to fill these gaps.

3. Basic analysis

3.1. The airports demand functions

To focus on airport competition and privatization in multi-airport systems, in this paper we assume away airline market power and strategic interactions at airports. As a result, air ticket price is exogenously given and is treated as constant as far as airports are concerned. The airport demand structure for atomistic carriers has been discussed by many previous studies (e.g., Zhang and Zhang, 2003; Oum et al., 2004; Xiao et al., 2013) in the context of a single-airport system. Following their basic specification, passenger demand faced by each airport in a two-airport region is modeled as a general function of "full prices" at both competing airports as in (1):

$$q_i = q_i\left(\rho_i, \rho_j\right); \quad i,j = 1,2 \tag{1}$$

where ρ_i is the full price perceived by passengers at airport i (i = 1, 2), comprising per passenger service price (P_i) and congestion delay cost (D_i) at that airport. The latter is itself a function of the airport's demand (q_i) and capacity (K_i):

$$\rho_i = P_i + D_i(q_i, K_i); \qquad i = 1,2 \tag{2}$$

Further, we have $\frac{\partial q_i}{\partial \rho_i} < 0$ and $\frac{\partial q_i}{\partial \rho_j} > 0$ $(i,j = 1,2; i \neq j)$ with the latter inequality capturing the notion that the two airports are competing with each other for passengers of the region. Note that the two airports do not need to be perfect substitutes here, as is the case for most multi-airport systems in the U.S. (e.g., Bonnefoy and Hansman, 2007).

[1] There are also several studies dealing with multi-airports, but these airports form an origin–destination pair and hence are complementary to each other rather than substitutes. For example, Basso (2008) examined the impact of privatization on pricing and capacity in such a two-airport system. He found that a system of profit-maximizing airports would overcharge for the congestion externality and restrict capacity investments; yet a system of welfare maximizing airports with cost recovery constraints achieve similar levels of congestion but less demand contraction.

The demand and capacity of an airport are measured by the number of passengers and the maximum number of serviced passengers per unit time interval, respectively. Even though airport capacity may be considered lumpy and indivisible (discrete capacity) since, for example, a new runway or terminal increases capacity in jumps (e.g., Oum and Zhang, 1990; Zhang and Zhang, 2003), we follow the assumption made in the majority of the previous research (e.g., De Borger and Van Dender, 2006; Basso and Zhang, 2007b, 2008; Zhang and Zhang, 2010) that D_i is differentiable in q_i and K_i, and

$$\frac{\partial D_i}{\partial q_i} > 0,\ \frac{\partial^2 D_i}{\partial q_i^2} > 0,\ \frac{\partial D_i}{\partial K_i} < 0,\ \frac{\partial^2 D_i}{\partial q_i \partial K_i} < 0; \quad i = 1,2 \tag{3}$$

Eq. (3) indicates that increasing demand will increase congestion delay cost while expanding capacity will reduce congestion delay cost and that these effects will be much more pronounced when there is more demand.

As indicated above, the demand structure presented here implicitly assumes the presence of atomistic carriers. As analytically proven by Basso and Zhang (2008), directly specifying airport demand—instead of deriving it from airlines' strategic behavior and passengers' demands in a vertical structure (e.g., Daniel, 1995; Brueckner, 2002), which involves (upstream) airports, (mid-stream) airlines, and (downstream) passengers—corresponds to a vertical structure only when the airline market is perfectly competitive (atomistic airlines). Although accounting for the vertical structure might be a more reasonable approach to airport pricing and capacity investment when carriers have market power and behave strategically, doing this would require detailed knowledge about how competition takes place in the airline market, and about airlines' costs and demands. Furthermore, explicit analytical modeling of airline competition often requires additional assumptions about aircraft size and load factor, and may make it more challenging to obtain analytical insights because of the additional layer of complexity (i.e., airline strategic behavior), especially in a general setting.[2] For these concerns, in this study we choose to directly specify airport demand, which also allows us to address our main research issue of airport competition and privatization in multi-airport systems. As further argued by Basso and Zhang (2007a, 2008), when studying complicated models that involve concession and competition between airports, both approaches (i.e., with and without considering airline behavior) may be used.

3.2. Baseline: two airports under a single public agency (social optimum)

Here, the public airport agency chooses the capacities and service charges of airports, $\boldsymbol{K} \equiv (K_1, K_2)$ and $\boldsymbol{P} \equiv (P_1, P_2)$, to maximize social welfare (*SW*), which, due to the existence of two stakeholders in our model, i.e. passengers and airports, is the sum of consumers' surplus and airports' profits. We use $U = U(q_1, q_2)$ to denote the utility of a representative consumer multiplied by the consumer population in the metropolitan region. The objective function $SW(\boldsymbol{P}, \boldsymbol{K})$ can be expressed as:

$$Max_{(\boldsymbol{P},\boldsymbol{K})} SW(\boldsymbol{P},\boldsymbol{K}) = \underbrace{U - \sum\nolimits_{i=1,2}(\rho_i q_i - v_i q_i)}_{Consumers' surplus} + \underbrace{\sum\nolimits_{i=1,2}[(P_i + h_i - c_i)q_i - r_i K_i]}_{Airports' profits} = U + \sum_{i=1,2}[(h_i + v_i - c_i - D_i)q_i - r_i K_i] \tag{4}$$

where r_i is the unit cost of capacity for airport i; h_i the average profit of airport i from concession services for each passenger; v_i the associated consumer surplus from concession. Both h_i and v_i are assumed to be positive and exogenously determined. Moreover, we assume that each airport has a constant marginal operating cost c_i for each passenger.

Taking (1) and (2) into account and using the chain rule, the marginal impacts of service charges on airports' demands can be derived in (5.1)–(5.3), the technical details of which are deferred to Appendix. The parenthesis in the numerator of (5.1) is nonnegative, because the absolute value of each price elasticity is larger than its respective cross-price elasticity. In addition, H is always positive.[3] The signs of (5.1) and (5.2) then can be determined, by further noting that $\frac{\partial q_i}{\partial \rho_i} < 0$ and $\frac{\partial q_i}{\partial \rho_j} > 0$ $(i,j = 1,2; i \neq j)$. As expected, an increase in the service charge of an airport will reduce demand at that airport while augmenting demand at the competing airport.

$$\frac{\partial q_i}{\partial P_i} = \frac{\frac{\partial q_i}{\partial \rho_i} - \frac{\partial D_j}{\partial q_j}\left(\frac{\partial q_i}{\partial \rho_i}\frac{\partial q_j}{\partial \rho_j} - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i}\right)}{H} < 0; \qquad i,j = 1,2 \tag{5.1}$$

$$\frac{\partial q_i}{\partial P_j} = \frac{\frac{\partial q_i}{\partial \rho_j}}{H} > 0; \qquad i,j = 1,2 \tag{5.2}$$

where

[2] For example, a common assumption in the vertical-structure literature is that all the flights use identical aircraft and have the same load factor (see, e.g., the survey paper by Zhang and Czerny (2012)), then yielding that each flight has an equal number of passengers, which facilitates the analysis. As pointed out by an anonymous referee, this assumption may have large implications on the results, as shown in, e.g., Daniel (2011), Vaze and Barnhart (2012a), Yan and Winston (2014) and Wei and Hansen (2007).

[3] In (5.3), $\frac{\partial q_i}{\partial \rho_i}\frac{\partial D_i}{\partial q_i} < 0$; $\frac{\partial D_i}{\partial q_i}\frac{\partial D_j}{\partial q_j} > 0$; and $\frac{\partial q_i}{\partial \rho_i}\frac{\partial q_j}{\partial \rho_j} - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i} > 0$ $(i,j = 1,2)$ as part of the second-order condition for maximization (i.e., the negative definiteness of the Hessian). The last one follows the discussions of the sign for (5.1).

$$H = \left(1 - \frac{\partial q_i}{\partial \rho_i}\frac{\partial D_i}{\partial q_i}\right)\left(1 - \frac{\partial q_j}{\partial \rho_j}\frac{\partial D_j}{\partial q_j}\right) - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i}\frac{\partial D_i}{\partial q_i}\frac{\partial D_j}{\partial q_j} = 1 - \frac{\partial q_i}{\partial \rho_i}\frac{\partial D_i}{\partial q_i} - \frac{\partial q_j}{\partial \rho_j}\frac{\partial D_j}{\partial q_j} + \frac{\partial D_i}{\partial q_i}\frac{\partial D_j}{\partial q_j}\left(\frac{\partial q_i}{\partial \rho_i}\frac{\partial q_j}{\partial \rho_j} - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i}\right) > 0;$$
$$i,\ j = 1,2 \tag{5.3}$$

The marginal effects of capacity expansions on airports' demands can also be derived, as shown in (6.1)-(6.2). Derivation details are provided in Appendix. The signs of (6.1) and (6.2) can be easily determined using (3) and (5.1)-(5.2): increasing airport capacity invites more demand; however, demand at the rival airport will be discouraged, which is consistent with intuition.

$$\frac{\partial q_i}{\partial K_i} = \frac{\partial q_i}{\partial P_i}\frac{\partial D_i}{\partial K_i} > 0; \qquad i = 1,2 \tag{6.1}$$

$$\frac{\partial q_i}{\partial K_j} = \frac{\partial q_i}{\partial P_j}\frac{\partial D_j}{\partial K_j} < 0; \qquad i,j = 1,2 \tag{6.2}$$

The optimal airport service charge and capacity are obtained by taking the first order derivatives of (4) with respect to $P_1,\ P_2,\ K_1,\ K_2$. Results are shown in Eqs. (7) and (8). Details of the derivations can be found in Appendix. Interestingly, the capacity rule for each airport is the same as for the case of a single public airport in a metropolitan area (Oum et al., 2004; Zhang and Zhang, 2010): the optimal capacity is set so that the marginal benefit of capacity expansion in terms of delay reduction equals the marginal cost of capacity. This finding also generalizes the results in Basso and Zhang (2007b) and De Borger and Van Dender (2006) for multi-airport regions.

For pricing rules, Eq. (8) shows that the optimal service charge at one airport consists of marginal social costs: the sum of marginal operating cost c_i and marginal congestion cost $q_i \frac{\partial D_i}{\partial q_i}$, and two markdown terms: the average concession profit h_i and the associated consumer surplus v_i. The markdown terms highlight the role of concession activities in reducing aeronautical service charge. Note that the social planner may set the optimal level of service price for each airport either below or above the airport break-even level (which is $c_i + r_i - h_i$), with the difference being $\left(q_i \frac{\partial D_i}{\partial q_i} - v_i - r_i\right),\ i = 1,2$. Similar to the optimal capacity rule, the optimal service charge for each airport is a function of only the airport's own characteristics $\left(c_i, h_i, v_i,\ \text{and}\ q_i \frac{\partial D_i}{\partial q_i}\right)$, although the term $q_i \frac{\partial D_i}{\partial q_i}$ suggests implicit dependence on the other airport's capacity and pricing levels.

$$-q_i \frac{\partial D_i}{\partial K_i} = r_i; \qquad i = 1,2 \tag{7}$$

$$P_i^* = c_i - h_i - v_i + q_i \frac{\partial D_i}{\partial q_i}; \qquad i = 1,2 \tag{8}$$

4. Capacity and pricing choice under privatization

Having specified the general demand framework for each airport and examined the base case of a single public authority that controls both airports, in this section we investigate capacity and pricing rules under privatization. Specifically, we consider three scenarios in which the social planner decides to privatize one or both airports through partial or complete privatization: (1) one airport becomes privatized, whereas the other remains public, i.e., public–private duopoly; (2) both airports become privatized by handing them over to two distinct private agencies, i.e., private–private duopoly; and (3) both airports become privatized and controlled by a private monopolist.

4.1. Scenario 1: one airport is privatized (public–private duopoly)

In the first privatization scenario, a pure public, centralized airport system is partially privatized to form a public–private duopoly in the metropolitan region. Herein we consider airport 2 as the one handed over to the private sector. Consequently, the public agency concentrates on maximizing social welfare from servicing passengers at airport 1, whereas the newly privatized airport seeks to maximize profit.

We consider both sequential and simultaneous decisions on capacity and service price.[4] In the sequential decision case, capacities are determined prior to service prices, whose choice hinges upon the capacity decisions taken. The sequential decision setting seems natural in that service price is easier to adjust than capacity, and is particularly valid when capacity improvement at an airport is easily observable by the competitor. The simultaneous decision setting holds when decisions of an airport are unobservable by its competitor (although the actual decisions may be made sequentially). The simultaneous

[4] Note that in the two monopoly cases – a public monopoly (considered in Section 3) or a private monopoly to be discussed below – both the sequential and simultaneous decisions lead to the same results.

decision setting can be further justified as the degree of commercialization, privatization, and deregulation of airports increases (Basso and Zhang, 2007b).

4.1.1. Sequential capacity and pricing decisions

Under the sequential decision setting, the airports' pricing and capacity choice is modeled in a two-stage game. At the first stage, the two airports simultaneously choose their capacities, given which service prices are determined at the second stage. As mentioned above, airport 1 maximizes welfare; whereas airport 2 maximizes profit. We solve for the sub-game perfect Nash equilibrium of the game in an order that is reverse of the actual decision process, by first looking at the service price determination conditional on capacities and then capacity choice. The objective functions of the two airports at the second stage are therefore:

$$Max_{P_1} SW_1(\boldsymbol{P})|\boldsymbol{K} = \underbrace{\int_0^{q_1} \rho_1(y, q_2)dy - \rho_1 q_1 + v_1 q_1}_{Consumer\ surplus} + \underbrace{(P_1 + h_1 - c_1)q_1 - r_1 K_1}_{Airport\ profit}$$
$$= \int_0^{q_1} \rho_1(y, q_2)dy + (h_1 + v_1 - c_1 - D_1)q_1 - r_1 K_1 \tag{9}$$

$$Max_{P_2} \pi_2(\boldsymbol{P})|\boldsymbol{K} = (P_2 + h_2 - c_2)q_2 - r_2 K_2 \tag{10}$$

The optimal service charges are derived by taking the first-order conditions as in (11) and (12):

$$\frac{\partial SW_1}{\partial P_1} = \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_1}\frac{\partial q_1}{\partial P_1} + \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\frac{\partial q_2}{\partial P_1} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_1} = 0 \tag{11}$$

$$\frac{\partial \pi_2}{\partial P_2} = (P_2 + h_2 - c_2)\frac{\partial q_2}{\partial P_2} + q_2 = 0 \tag{12}$$

Replacing the expressions of marginal pricing effects on demands, $\frac{\partial q_i}{\partial P_i}$ and $\frac{\partial q_i}{\partial P_j}$ $(i, j = 1, 2)$ by (5.1) and (5.2), and performing some algebra, the optimal pricing rules can be obtained as shown in (13) and (14). Details of the derivation are again deferred to Appendix.

$$P_1 = \left(c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1}\right) - \underbrace{\frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}}_{-}\underbrace{\frac{\varepsilon_{21}\rho_2 q_2}{\varepsilon_1 \rho_2 q_1 - (\varepsilon_1\varepsilon_2 - \varepsilon_{12}\varepsilon_{21})q_1 q_2\frac{\partial D_2}{\partial q_2}}}_{-} \tag{13}$$

$$P_2 = \left(c_2 - h_2 + q_2\frac{\partial D_2}{\partial q_2}\right) - \frac{1}{\underbrace{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}}_{-}} \tag{14}$$

For the remaining public airport 1, the optimal service price consists of marginal operating cost c_1, marginal congestion cost $q_1\frac{\partial D_1}{\partial q_1}$, and three mark-down terms: concession profit h_1, concession consumer surplus v_1, and the last term in (13). This last term is the product of two negative terms:

(1) The marginal effect of demand at airport 2 on the total willingness-to-pay of all passengers at airport 1, which is negative because airports provide substitute services. More specifically, an increase in the demand of airport 2 will shift the demand curve of airport 1 inwards, leading to a reduction in the total passenger willingness-to-pay at airport 1. Clearly, the absolute value of this marginal effect increases as the two airports' services become more substitutable.
(2) A fraction that accounts for the effect of elasticities on service price of the public airport. Since full prices, demands, and marginal delay effect of demand (according to Eq. (3)) are positive, and self elasticities are negative with larger absolute values than cross elasticities, the numerator of the fraction is positive while the denominator is negative. Therefore, the second term is negative. The fraction will decrease in absolute value if airports' demands are more elastic to their own prices, or cross demand elasticities become smaller, or the privatized airport is more congestible (i.e., larger $\frac{\partial D_2}{\partial q_2}$).

For the privatized airport, the optimal price will not take into account consumer surplus from concession. However, a markup (Appendix (equation (A-19)) shows why this term has a positive sign) is added to the service price (last term in (14)) which depends on the congestion at the public airport, passenger full price, and the elasticities of demand. Note that $1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1} > 0$, $q_1\frac{\partial D_1}{\partial q_1} > 0$, and $\frac{\varepsilon_2}{\rho_2} < 0$. Therefore, this markup term decreases if either airport's demand becomes more elastic to its own price, but increases with larger cross elasticities.

The optimal pricing rules in (13) and (14) represent the implicit airport price response function given capacity, i.e., $P_1^R(P_2, K_1, K_2)$ and $P_2^R(P_1, K_1, K_2)$, with the superscript R denoting that they are response functions. The Nash equilibrium service prices for known capacities, i.e., $P_1^*(K_1, K_2)$ and $P_2^*(K_1, K_2)$, can then be found at the intersection of the two price response functions.

Moving backward to the first stage, each airport determines the capacity to maximize its objective with service prices expressed as response functions of the optimal capacities:

$$Max_{K_1}\boldsymbol{SW_1}(\boldsymbol{K}) = \int_0^{q_1} \rho_1(y, q_2)dy + (h_1 + v_1 - c_1 - D_1)q_1 - r_1K_1 \tag{15}$$

$$Max_{K_2}\boldsymbol{\pi_2}(\boldsymbol{K}) = (P_2^* + h_2 - c_2)q_2 - r_2K_2 \tag{16}$$

Taking the derivatives of the objective functions with respect to $K_1,\ K_2$ at the first stage yields:

$$\begin{aligned}\frac{\partial SW_1}{\partial K_1} &= \rho_1 \frac{dq_1}{dK_1} + \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\frac{dq_2}{dK_1} + (h_1 + v_1 - c_1 - D_1)\frac{dq_1}{dK_1} - q_1\left(\frac{\partial D_1}{\partial q_1}\frac{dq_1}{dK_1} + \frac{\partial D_1}{\partial K_1}\right) - r_1 = 0 \\ &= \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\frac{dq_2}{dK_1} + \left(P_1^* + h_1 + v_1 - c_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{dq_1}{dK_1} - q_1\frac{\partial D_1}{\partial K_1} - r_1 = 0\end{aligned} \tag{17}$$

$$\frac{\partial \pi_2}{\partial K_2} = (P_2^* + h_2 - c_2)\frac{dq_2}{dK_2} + q_2\frac{\partial P_2^*}{\partial K_2} - r_2 = 0 \tag{18}$$

Since equilibrium service price of each airport is now a function of airports' capacities, the marginal effect of capacity improvement on airport demand above cannot be expressed like (6.1) and (6.2). Instead, we derive these marginal effects in Appendix and present the results in (19). Each marginal effect consists of the direct effect (first terms in (19.1)), by holding service prices constant, and the indirect effect via strategic Nash equilibrium price adjustments at the second stage of the game (second terms in (19.1) and (19.2)). The signs of these marginal impacts are indeterminate in general.

$$\frac{dq_i}{dK_i} = \frac{\partial q_i}{\partial K_i} + \frac{\frac{\partial P_j^*}{\partial K_i}\frac{\partial q_i}{\partial \rho_j} + \frac{\partial P_i^*}{\partial K_i}\left(\frac{\partial q_i}{\partial \rho_i} - \frac{\partial D_j}{\partial q_j}\left(\frac{\partial q_i}{\partial \rho_i}\frac{\partial q_j}{\partial \rho_j} - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i}\right)\right)}{H};\ i, j = 1, 2 \tag{19.1}$$

$$\frac{dq_i}{dK_j} = \frac{\partial q_i}{\partial K_j} + \frac{\frac{\partial P_j^*}{\partial K_j}\frac{\partial q_i}{\partial \rho_j} + \frac{\partial P_i^*}{\partial K_j}\left(\frac{\partial q_i}{\partial \rho_i} - \frac{\partial D_j}{\partial q_j}\left(\frac{\partial q_i}{\partial \rho_i}\frac{\partial q_j}{\partial \rho_j} - \frac{\partial q_i}{\partial \rho_j}\frac{\partial q_j}{\partial \rho_i}\right)\right)}{H};\ i, j = 1, 2 \tag{19.2}$$

Since each airport chooses its capacity by taking Nash equilibrium service price into consideration, the objective functions (15) and (16), which are known as value functions, can be reformulated by substituting the Nash equilibrium service prices into the objective functions of the second stage, i.e., $\boldsymbol{SW_1}(\boldsymbol{K}) = SW_1(\boldsymbol{P}^*(\boldsymbol{K}), \boldsymbol{K})$ and $\boldsymbol{\pi_2}(\boldsymbol{K}) = \pi_2(\boldsymbol{P}^*(\boldsymbol{K}),\ \boldsymbol{K})$. Applying the envelope theorem, the Nash equilibrium capacities are determined by the following first-order conditions:

$$\frac{\partial \boldsymbol{SW_1}(\boldsymbol{K})}{\partial K_1} = \underbrace{\frac{\partial SW_1}{\partial P_1}}_{0}\frac{\partial P_1^*}{\partial K_1} + \frac{\partial SW_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1} + \frac{\partial SW_1}{\partial K_1} = \frac{\partial SW_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1} + \frac{\partial SW_1}{\partial K_1} = 0 \tag{20}$$

$$\frac{\partial \boldsymbol{\pi_2}(\boldsymbol{K})}{\partial K_2} = \frac{\partial \pi_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \underbrace{\frac{\partial \pi_2}{\partial P_2}}_{0}\frac{\partial P_2^*}{\partial K_2} + \frac{\partial \pi_2}{\partial K_2} = \frac{\partial \pi_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \frac{\partial \pi_2}{\partial K_2} = 0 \tag{21}$$

The first term in (20) and the second term in (21) are equal to zero according to the first-order conditions in (11) and (12). Substituting (17) and (18), $\rho_1\frac{\partial q_1}{\partial P_2} + \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\frac{\partial q_2}{\partial P_2} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_2}$, and $(P_2^* + h_2 - c_2)\frac{\partial q_2}{\partial P_1}$ for $\frac{\partial SW_1}{\partial K_1}$, $\frac{\partial \pi_2}{\partial K_2}$, $\frac{\partial SW_1}{\partial P_2}$ and $\frac{\partial \pi_2}{\partial P_1}$ above leads to the following optimal capacity rules:

$$\begin{aligned}&[(P_1^* + h_1 - c_1) + v_1]\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right) + \frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\left(\frac{\partial q_2}{\partial K_1} + \frac{\partial q_2}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right) - q_1\left[\frac{\partial D_1}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)\right] \\ &= r_1\end{aligned} \tag{22}$$

$$(P_2^* + h_2 - c_2)\left(\frac{\partial q_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \frac{\partial q_2}{\partial K_2}\right) = r_2 \tag{23}$$

For the privatized airport, the optimal capacity rule (23) equates marginal capacity cost r_2 with the marginal profit effect of capacity $(P_2^* + h_2 - c_2)\left(\frac{\partial q_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \frac{\partial q_2}{\partial K_2}\right)$, where $P_2^* + h_2 - c_2$ is the profit gain from serving one additional passenger; $\left(\frac{\partial q_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \frac{\partial q_2}{\partial K_2}\right)$ represents the response of q_2 to one unit increase in K_2. It is worth noting that this demand response is composed of direct effect $\left(\frac{\partial q_2}{\partial K_2}\right)$ and indirect effect $\left(\frac{\partial q_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2}\right)$, the latter through airport 1's service price response to K_2.

For the remaining public airport, it is not surprising that the optimal capacity rule (22) equates marginal capacity cost r_1 to the marginal welfare effects of capacity, which consist of:

(i) airport profit change $(P_1^* + h_1 - c_1)\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)$;
(ii) passenger welfare gains through concession $v_1\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)$;
(iii) passenger welfare gains through shift of the demand curve $\frac{\partial\int_0^{q_1}\rho_1(y,q_2)dy}{\partial q_2}\left(\frac{\partial q_2}{\partial K_1} + \frac{\partial q_2}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)$;
(iv) airport delay reduction $-q_1\left[\frac{\partial D_1}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)\right]$. The marginal delay change consists of direct effect $\left(\frac{\partial D_1}{\partial K_1}\right)$ and indirect effect through change in q_1.

4.1.2. Simultaneous capacity and pricing decisions

We now turn to public–private duopoly with simultaneous capacity and pricing decisions. The objective functions for the two airports are as follows:

$$Max_{(P_1,K_1)}SW_1(\boldsymbol{P},\boldsymbol{K}) = \int_0^{q_1}\rho_1(y,q_2)dy + (h_1 + v_1 - c_1 - D_1)q_1 - r_1K_1 \tag{24}$$

$$Max_{(P_2,K_2)}\pi_2(\boldsymbol{P},\boldsymbol{K}) = (P_2 + h_2 - c_2)q_2 - r_2K_2 \tag{25}$$

The first-order conditions are:

$$\frac{\partial SW_1}{\partial P_1} = \rho_1\frac{\partial q_1}{\partial P_1} + \frac{\partial\int_0^{q_1}\rho_1(y,q_2)dy}{\partial q_2}\frac{\partial q_2}{\partial P_1} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_1} = 0 \tag{26.1}$$

$$\frac{\partial SW_1}{\partial K_1} = \rho_1\frac{\partial q_1}{\partial K_1} + \frac{\partial\int_0^{q_1}\rho_1(y,q_2)dy}{\partial q_2}\frac{\partial q_2}{\partial K_1} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial K_1} - q_1\frac{\partial D_1}{\partial K_1} - r_1 = 0 \tag{26.2}$$

$$\frac{\partial \pi_2}{\partial P_2} = (P_2 + h_2 - c_2)\frac{\partial q_2}{\partial P_2} + q_2 = 0 \tag{27.1}$$

$$\frac{\partial \pi_2}{\partial K_2} = (P_2 + h_2 - c_2)\frac{\partial q_2}{\partial K_2} - r_2 = 0 \tag{27.2}$$

After some algebraic manipulation, the optimal capacity and pricing rules (28)–(31) are obtained.[5] Not surprisingly, the optimal pricing rule 30, 31 are identical to the price response expressions in the sequential game 13, 14, but capacity rules are different. Optimal airport capacity is determined in a way such that marginal cost of capacity equals marginal delay reduction benefits, which is the same as in the one public agency case (Eq. (7)).

$$-q_1\frac{\partial D_1}{\partial K_1} = r_1 \tag{28}$$

$$-q_2\frac{\partial D_2}{\partial K_2} = r_2 \tag{29}$$

$$P_1^* = c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1} - \frac{\partial\int_0^{q_1}\rho_1(y,q_2)dy}{\partial q_2}\frac{\varepsilon_{21}\rho_2q_2}{\varepsilon_1\rho_2q_1 - (\varepsilon_1\varepsilon_2 - \varepsilon_{12}\varepsilon_{21})q_1q_2\frac{\partial D_2}{\partial q_2}} \tag{30}$$

$$P_2^* = \left(c_2 - h_2 + q_2\frac{\partial D_2}{\partial q_2}\right) - \frac{1}{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1-\frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}} \tag{31}$$

[5] Note in (6.1) that $\frac{\partial q_i}{\partial K_i} = \frac{\partial q_i}{\partial P_i}\frac{\partial D_i}{\partial K_i}(i,j = 1,2)$. Then (28) can be easily derived by comparing (26.1) and (26.2). Likewise for (29). Derivation of (30) and (31) follows the same process as for (13) and (14).

It should be noted that although both the public and the private airports consider the same capacity rules as in the pre-privatization case, their capacity levels are not equal since they hold at different service price and demand levels.

4.2. Scenario 2: both airports are privatized and owned by competing agencies (private–private duopoly)

In the second privatization scenario, we investigate the transition to a private–private duopoly, in which the two airports are handed over to two private firms, each choosing capacity and service price to maximize its profit. Similar to the public–private duopoly, both sequential and simultaneous decision settings are examined.

4.2.1. Sequential capacity and pricing decisions

The derivation of capacity and pricing rules follows the same rationale as in Section 4.1.1. At the stage of determining optimal service price, the objective functions for the two airports and their first-order conditions are:

$$Max_{P_i}\pi_i(\boldsymbol{P})|\boldsymbol{K} = (P_i + h_i - c_i)q_i - r_iK_i; \qquad i = 1,2 \tag{32}$$

$$\frac{\partial \pi_i}{\partial P_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial P_i} + q_i = 0; \qquad i = 1,2 \tag{33}$$

By substituting (5.1) and (5.2) for the marginal effect of service price on demand and performing some algebra, the pricing rules are derived as expressed in (34), which are the same as for the privatized airport under partial privatization ((14) and (31)). More specifically, the optimal service charges at both privatized airports will equal the marginal social cost minus concession profit, $c_1 - h_1 + q_1\frac{\partial D_1}{\partial q_1}$, together with a markup (the last term in (34)) which depends on congestion at the rival airport, passenger full price, and demand elasticities.

$$P_i = \left(c_i - h_i + q_i\frac{\partial D_i}{\partial q_i}\right) - \frac{1}{\frac{\varepsilon_i}{\rho_i} + \frac{\varepsilon_{ij}\varepsilon_{ji}}{\rho_i\rho_j}\frac{q_j\frac{\partial D_j}{\partial q_j}}{1 - \frac{\varepsilon_j}{\rho_j}q_j\frac{\partial D_j}{\partial q_j}}}; \qquad i,j = 1,2 \tag{34}$$

Again, the pricing rule (34) implicitly defines airport price response function given capacity $P_1^R(P_2, K_1, K_2)$ and $P_2^R(P_1, K_1, K_2)$. The Nash equilibrium service prices for known capacities $P_1^*(K_1, K_2)$ and $P_2^*(K_1, K_2)$ can be identified at the intersection of the two price response functions.

The second-stage problem and its first order conditions are:

$$Max_{K_i}\boldsymbol{\pi}_i(\boldsymbol{K}) = (P_i^* + h_i - c_i)q_i - r_iK_i; \qquad i = 1,2 \tag{35}$$

$$\frac{\partial \pi_i}{\partial K_i} = (P_i^* + h_i - c_i)\frac{dq_i}{dK_i} + q_i\frac{\partial P_i^*}{\partial K_i} - r_i = 0; \qquad i = 1,2 \tag{36}$$

where $\boldsymbol{\pi_i}(\boldsymbol{K}) \equiv \pi_i(\boldsymbol{P}^*(\boldsymbol{K}),\ \boldsymbol{K}),\ i,j = 1,2$. Following the envelope theorem, the Nash equilibrium capacities can be determined by the following first-order conditions:

$$\frac{\partial \boldsymbol{\pi_i}(\boldsymbol{K})}{\partial K_i} = \underbrace{\frac{\partial \pi_i}{\partial P_i}}_{0}\frac{\partial P_i^*}{\partial K_i} + \frac{\partial \pi_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i} + \frac{\partial \pi_i}{\partial K_i} = \frac{\partial \pi_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i} + \frac{\partial \pi_i}{\partial K_i} = 0; \qquad i,j = 1,2 \tag{37}$$

which leads to the optimal airport capacity rule by substituting $(P_i^* + h_i - c_i)\frac{\partial q_i}{\partial P_j}$ and $(P_i + h_i - c_i)\frac{\partial q_i}{\partial K_i} - r_i$ for $\frac{\partial \pi_i}{\partial P_j}$ and $\frac{\partial \pi_i}{\partial K_i}$:

$$(P_i^* + h_i - c_i)\left(\frac{\partial q_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i} + \frac{\partial q_i}{\partial K_i}\right) = r_i; \qquad i,j = 1,2 \tag{38}$$

It is interesting to observe that (38) is identical to the capacity rule for the privatized airport under partial privatization (23), although their realized capacity levels are different since they hold at different service price and demand levels. Therefore, a privatized airport's capacity under sequential decision duopoly will be set such that the marginal capacity cost is equal to the marginal capacity effect on profit, regardless of whether the other airport remains pubic or not.

4.2.2. Simultaneous capacity and pricing decisions

When capacity and service price are simultaneously determined, the objective functions of the two privatized airports are expressed as:

$$Max_{(P_i,K_i)}\pi_i(\boldsymbol{P}, \boldsymbol{K}) = (P_i + h_i - c_i)q_i - r_iK_i; \qquad i = 1,2 \tag{39}$$

Taking first-order conditions: $\frac{\partial \pi_i}{\partial P_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial P_i} + q_i = 0$ and $\frac{\partial \pi_i}{\partial K_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial K_i} - r_i = 0,\ i = 1,2$, we derive capacity and pricing rules as in (40) and (41). Same as in Section 4.2.1, these rules are invariant for a privatized airport whether the other airport remains public.

$$-q_i \frac{\partial D_i}{\partial K_i} = r_i; \qquad i = 1,2 \tag{40}$$

$$P_i^* = \left(c_i - h_i + q_i \frac{\partial D_i}{\partial q_i}\right) - \frac{1}{\frac{\varepsilon_i}{\rho_i} + \frac{\varepsilon_{ij}\varepsilon_{ji}}{\rho_i\rho_j} \frac{q_j \frac{\partial D_j}{\partial q_j}}{1 - \frac{\varepsilon_j}{\rho_j} q_j \frac{\partial D_j}{\partial q_j}}}; \qquad i,j = 1,2 \tag{41}$$

4.3. Scenario 3: both airports are privatized and owned by a single private agency (private monopoly)

When the two-airport system becomes a privatized monopoly, the objective is to maximize the collective profit from the two airports, by choosing the appropriate service price and capacity levels:

$$Max_{(\boldsymbol{P},\boldsymbol{K})} \pi(\boldsymbol{P}, \boldsymbol{K}) = \sum_{i=1,2} [(P_i + h_i - c_i)q_i - r_i K_i] \tag{42}$$

The first-order conditions are: $\frac{\partial \pi}{\partial P_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial P_i} + (P_j + h_j - c_j)\frac{\partial q_j}{\partial P_i} + q_i = 0$ and $\frac{\partial \pi}{\partial K_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial K_i} + (P_j + h_j - c_j)\frac{\partial q_j}{\partial K_i} - r_i = 0,\ i = 1,2$. Substituting (5.1), (5.2), (6.1) and (6.2) for the marginal effects of service charge and capacity on airport demand above yields the optimal capacity rule (43). Further manipulation leads to the optimal pricing rule (44), with derivation detailed in Appendix.

$$-q_i \frac{\partial D_i}{\partial K_i} = r_i;\ i = 1,2 \tag{43}$$

$$P_i^* = c_i - h_i + q_i \frac{\partial D_i}{\partial q_i} + \underbrace{\frac{q_i\rho_i\varepsilon_j - q_j\rho_j\varepsilon_{ji}}{q_i(\varepsilon_{ij}\varepsilon_{ji} - \varepsilon_i\varepsilon_j)}}_{+}; \qquad i,j = 1,2 \tag{44}$$

Comparison of (43) with (7) reveals that, whether it is a private monopoly or a public monopoly (considered in Section 3), the optimal level of capacity is set such that the marginal capacity cost equals the marginal benefit of delay reduction. Eq. (43) reproduces the results in De Borger and Van Dender (2006), but in a generalized setting.

Eq. (44) shows that the optimal service price at each airport is equal to the marginal social cost minus concession profit. The monopoly power further results in a markup in service price which is the last term in (44).[6] It is easy to verify that the markup decreases if the airport's demand becomes more elastic to its own service price, but increases with larger cross elasticities.

5. Discussions

Building upon the analytical results derived above, this section offers a comparative examination of airport capacity and pricing choice with and without airport privatization. The optimal capacity and pricing rules under the baseline and three privatization scenarios are summarized in Table 1. In what follows, a synthetic comparison of airport pricing and capacity rules is conducted, which generalizes the findings in the literature (De Borger and Van Dender, 2006; Basso and Zhang, 2007b) for full privatization under duopoly and monopoly, but without relying on specific demand and delay functions. It is worth noting that these two previous studies both considered delays as a linear function of demand–capacity ratio, which was considered a poor assumption (Brueckner, 2002; Vaze and Barnhart, 2012a). In addition, a more unified framework for capacity and pricing rules is established across all the organization structures considered in a two-airport system.

5.1. Airport service price

Prior to privatization, service price at an airport is equal to the net marginal social cost (MSC_i), which is the sum of marginal operating cost (c_i) and congestion cost ($q_i \frac{\partial D_i}{\partial q_i}$), purged of marginal (which in our case is also average) welfare gains from concession consisting of profit (h_i) and consumer surplus (v_i). After privatization takes place, a privatized airport will not consider consumer surplus from concession and tends to increase the service price across all privatization scenarios considered. There exist two ways to determine the price markup, one for duopoly and one for monopoly. While a number of factors such as demand, demand elasticities, passenger full price, and airport delay are involved, the two ways to mark up service price are similar in that the markup decreases with an airport's own demand elasticity, and increases with cross elasticities.

On the other hand, the remaining public airport under the public–private monopoly tends to mark down the service price. The price markdown should be interpreted as the result of the competition pressure from the privatized airport.

The above discussions are synthesized in Proposition 1:

[6] Since full prices and demands are positive, and own price elasticities of demands are negative with larger absolute values than the positive cross elasticities of demands, both the numerator and the denominator of the last terms in (44) are negative. Thus, the entire terms are positive.

Table 1
Summary of capacity and pricing rules for each market structure in a two-airport metropolitan area.

Market structure	Pricing rule	Capacity rule
1. **Social optimum**	$P_i^* = (MSC_i - h_i) - v_i \ i = 1,2$	$-q_i \frac{\partial D_i}{\partial K_i} = r_i \ i = 1,2$
2. **Public–private duopoly**		
Sequential decisions	$P_1 = (MSC_1 - h_1) - v_1 - \underbrace{\frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2} \frac{\varepsilon_{21}\rho_2 q_2}{\varepsilon_1 \rho_2 q_1 - (\varepsilon_1\varepsilon_2 - \varepsilon_{12}\varepsilon_{21})q_1 q_2 \frac{\partial D_2}{\partial q_2}}}_{-}$	$[(P_1^* + h_1 - c_1) + v_1]\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right) + \frac{\partial \int_0^{q_1} \rho_1(y,q_2)dy}{\partial q_2}\left(\frac{\partial q_2}{\partial K_1} + \frac{\partial q_2}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right) - q_1[\frac{\partial D_1}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)] = r_1$
	$P_2 = (MSC_2 - h_2) - \underbrace{\frac{1}{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}}}_{+}$	$(P_2^* + h_2 - c_2)\left(\frac{\partial q_2}{\partial P_1}\frac{\partial P_1^*}{\partial K_2} + \frac{\partial q_2}{\partial K_2}\right) = r_2$
Simultaneous decisions	$P_1^* = (MSC_1 - h_1) - v_1 - \underbrace{\frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2} \frac{\varepsilon_{21}\rho_2 q_2}{\varepsilon_1 \rho_2 q_1 - (\varepsilon_1\varepsilon_2 - \varepsilon_{12}\varepsilon_{21})q_1 q_2 \frac{\partial D_2}{\partial q_2}}}_{-}$	$-q_1 \frac{\partial D_1}{\partial K_1} = r_1$
	$P_2^* = (MSC_2 - h_2) - \underbrace{\frac{1}{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}}}_{+}$	$-q_2 \frac{\partial D_2}{\partial K_2} = r_2$
3. **Private–private duopoly**		
Sequential decisions	$P_i = (MSC_i - h_i) - \underbrace{\frac{1}{\frac{\varepsilon_i}{\rho_i} + \frac{\varepsilon_{ij}\varepsilon_{ji}}{\rho_i\rho_j}\frac{q_j\frac{\partial D_j}{\partial q_j}}{1 - \frac{\varepsilon_j}{\rho_j}q_j\frac{\partial D_j}{\partial q_j}}}}_{+} \ i = 1,2$	$(P_i^* + h_i - c_i)\left(\frac{\partial q_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i} + \frac{\partial q_i}{\partial K_i}\right) = r_i \ i = 1,2$
Simultaneous decisions	$P_i^* = (MSC_i - h_i) - \underbrace{\frac{1}{\frac{\varepsilon_i}{\rho_i} + \frac{\varepsilon_{ij}\varepsilon_{ji}}{\rho_i\rho_j}\frac{q_j\frac{\partial D_j}{\partial q_j}}{1 - \frac{\varepsilon_j}{\rho_j}q_j\frac{\partial D_j}{\partial q_j}}}}_{+} \ i = 1,2$	$-q_i \frac{\partial D_i}{\partial K_i} = r_i \ i = 1,2$
4. **Private monopoly**	$P_i^* = (MSC_i - h_i) + \underbrace{\frac{q_i\rho_i\varepsilon_j - q_j\rho_j\varepsilon_{ji}}{q_i(\varepsilon_{ij}\varepsilon_{ji} - \varepsilon_i\varepsilon_j)}}_{+} \ i = 1,2$	$-q_i \frac{\partial D_i}{\partial K_i} = r_i \ i = 1,2$

Note: 1. $MSC_i = c_i + q_i \frac{\partial D_i}{\partial q_i}$; 2. Service prices with an asterisk in the superscript denote optimal pricing rules, while other prices represent response functions for prices.

Proposition 1. *In a public two-airport system, the service price for each airport prior to privatization equals its net marginal social cost. After partial or full privatization, a privatized airport tends to deviate upwards from the pre-privatization price; in contrast, the remaining public airport tends to decrease service price.*

5.2. Airport capacity

A greater extent of unanimity appears for the optimal airport capacity rules. Regardless of how airports are privatized, as long as capacity is determined simultaneously with service price, the capacity of an airport remains at the level where the marginal capacity cost equals marginal delay reduction benefits. Because price and capacity are chosen in one stage, the choice of capacity naturally does not account for the effect on equilibrium price. We formalize this as Proposition 2 below.

Proposition 2. *If service price and capacity decisions are made simultaneously, then capacity of an airport is always set such that marginal capacity cost equals marginal delay reduction. This rule is invariant to the ownership forms of a two-airport system.*

In contrast, when pricing and capacity are determined sequentially, determination of capacity must take account of its effect on pricing decisions, which further affects airport demand and profit. The consequent capacity rule becomes one such that marginal capacity cost equals marginal benefits with respect to one additional unit of capacity. The term "marginal benefit" refers to marginal profit when an airport becomes privatized, which is the product of unit profit from servicing one passenger $\left(P_i^* + h_i - c_i\right)$ and marginal demand change $\left(\frac{\partial q_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i} + \frac{\partial q_i}{\partial K_i}\right)$. Because the capacity decision at one airport now affects service price at the competing airport, which in turn affects demand of the airport under study, the marginal demand change through service price change of the competing airport $\left(\frac{\partial q_i}{\partial P_j}\frac{\partial P_j^*}{\partial K_i}\right)$ must be included. For the remaining public airport under public–private duopoly, the term "marginal benefit" captures marginal social welfare gains, which consists of not only airport profit increase, but also passenger welfare gains from concession $\left(v_1\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)\right)$, from demand curve shift due to demand change at the competing airport $\left(\frac{\partial \int_0^{q_1} \rho_1(y, q_2)dy}{\partial q_2}\left(\frac{\partial q_2}{\partial K_1} + \frac{\partial q_2}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)\right)$, and delay reduction $\left(q_1\left[\frac{\partial D_1}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial K_1} + \frac{\partial q_1}{\partial P_2}\frac{\partial P_2^*}{\partial K_1}\right)\right]\right)$. Again, the marginal effect on demand and delay of capacity has direct and indirect components. The above discussions lead to Proposition 3.

Proposition 3. *Under sequential decisions of service price and capacity, the privatized airport in a previously public two-airport system will set capacity at the level where marginal capacity cost equals marginal airport profit. This is true for both partial and full privatization. For the remaining public airport under partial privatization, the capacity level is set such that marginal capacity cost equals marginal social benefit.*

6. Concluding remarks

Multi-airport systems have been playing an increasingly important role in the airport sector. Given the growing importance of multi-airport regions in both traffic and the level of congestion in the air transportation system, and the worldwide trend of privatizing airports, this study fills the research gap by providing an analytical investigation of pricing and capacity choices of two airports that coexist in a metropolitan area. Airside congestion and concession operations are jointly considered, and our focus is on transformation from a pure public, centralized multi-airport system to three possible privatization scenarios. Our modeling results provide a more comprehensive and unified view on airport pricing and capacity investment under different organization forms. Over all the scenarios considered, it is found that the basic tenet of pricing and capacity investment remains valid, though in its varying forms: optimal service price equals marginal private or social cost with a markup or markdown depending on the ownership nature of an airport itself as well as its competitor; optimal capacity is set such that marginal capacity cost equates marginal benefits, which can be delay reduction, profit, or social welfare depending on whether capacity decisions are made prior to or simultaneously with pricing decisions, and on the ownership nature of an airport.

The paper also introduces several important avenues for future research. First, we treat airport capacity as differentiable in this paper. Airport capacities can be alternatively considered as lumpy and indivisible, especially when change in physical infrastructure is involved. Discretization of capacity would give rise to the issue of timing for capacity expansion, although this is not the focus of the present study. Second, as we, similar to the majority of previous studies, have not evaluated the impact of existing capacity levels on future capacity decisions, extension of the current research may look into how base capacity affects capacity investment decisions. Third, since a competitive carrier market is assumed in this paper, for airports with dominant carriers the model framework needs to be adjusted to reflect the specific airport–airline vertical structure. There can be two possible approaches: the analytical vertical-structure approach and an operations research based approach. While we expect the former to generate closed-form expressions for capacity and pricing rules, the difficulty of obtaining analytical insights is likely to increase as another layer of complexity (i.e., airline behavior) is added. The operations research based approach has the advantage of being flexible, but requires more modeling and computation efforts (e.g., Vaze and

Barnhart, 2012a; Vaze and Barnhart, 2012b). Fourth, one component that is not considered in the passenger full price is schedule delay. An increase in passenger demand leads to two consequences: greater congestion and more frequent services, with the latter reducing schedule delay of passengers. The obvious tradeoff between congestion delay and schedule delay would be very interesting and worthy of further investigation. Finally, given that airport privatization is gaining its momentum in many multi-airport metropolitan regions, empirical studies are warranted to check whether the theoretical results derived in the present study are consistent with the empirical evidence (e.g., Bilotkach et al., 2012).

Acknowledgements

We thank the three anonymous reviewers for very constructive comments which have significantly improved the structure and presentation of the paper.

Appendix A. Formula derivations

Proof of Eqs. (5.1)–(5.3): $\frac{\partial q_1}{\partial P_1}$ and $\frac{\partial q_2}{\partial P_1}$ are derived as follows.

$$\frac{\partial q_1}{\partial P_1}=\frac{\partial q_1(\rho_1,\rho_2)}{\partial P_1}=\frac{\partial q_1}{\partial \rho_1}\frac{\partial \rho_1}{\partial P_1}+\frac{\partial q_1}{\partial \rho_2}\frac{\partial \rho_2}{\partial P_1}=\frac{\partial q_1}{\partial \rho_1}\left(1+\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1}\right)+\frac{\partial q_1}{\partial \rho_2}\left(\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial P_1}\right) \tag{A-1}$$

$$\frac{\partial q_2}{\partial P_1}=\frac{\partial q_2(\rho_1,\rho_2)}{\partial P_1}=\frac{\partial q_2}{\partial \rho_1}\left(1+\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1}\right)+\frac{\partial q_2}{\partial \rho_2}\left(\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial P_1}\right)\Rightarrow\frac{\partial q_2}{\partial P_1}=\frac{1+\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1}}{1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}\frac{\partial q_2}{\partial \rho_1} \tag{A-2}$$

Plugging (A-2) into (A-1) yields $\frac{\partial q_1}{\partial P_1}$, as shown in (A-3):

$$\begin{aligned}\frac{\partial q_1}{\partial P_1}&=\frac{\partial q_1}{\partial \rho_1}\left(1+\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1}\right)+\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\frac{1+\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1}}{1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}\\&\Rightarrow\frac{\partial q_1}{\partial P_1}\underbrace{\left[\left(1-\frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)\left(1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right)-\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right]}_{H}=\frac{\partial q_1}{\partial \rho_1}\left(1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right)+\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\\&\Rightarrow\frac{\partial q_1}{\partial P_1}=\frac{\frac{\partial q_1}{\partial \rho_1}-\frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2}-\frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)}{H}\end{aligned} \tag{A-3}$$

Then, plugging (A-3) into (A-2) yields $\frac{\partial q_2}{\partial P_1}$. $\frac{\partial q_2}{\partial P_2}$ and $\frac{\partial q_1}{\partial P_2}$ can be similarly derived. □

Proof of Eqs. (6.1) and (6.2): When pricing and capacity choice are determined simultaneously, $\frac{\partial q_1}{\partial K_1}$ and $\frac{\partial q_2}{\partial K_1}$ are derived as follows.

$$\frac{\partial q_1}{\partial K_1}=\frac{\partial q_1(\rho_1,\rho_2)}{\partial K_1}=\frac{\partial q_1}{\partial \rho_1}\frac{\partial \rho_1}{\partial K_1}+\frac{\partial q_1}{\partial \rho_2}\frac{\partial \rho_2}{\partial K_1}=\frac{\partial q_1}{\partial \rho_1}\left(\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial K_1}+\frac{\partial D_1}{\partial K_1}\right)+\frac{\partial q_1}{\partial \rho_2}\left(\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial K_1}\right) \tag{A-4}$$

$$\frac{\partial q_2}{\partial K_1}=\frac{\partial q_2(\rho_1,\rho_2)}{\partial K_1}=\frac{\partial q_2}{\partial \rho_1}\left(\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial K_1}+\frac{\partial D_1}{\partial K_1}\right)+\frac{\partial q_2}{\partial \rho_2}\left(\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial K_1}\right)\Rightarrow\frac{\partial q_2}{\partial K_1}=\frac{\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial K_1}+\frac{\partial D_1}{\partial K_1}}{1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}\frac{\partial q_2}{\partial \rho_1} \tag{A-5}$$

Replacing $\frac{\partial q_2}{\partial K_1}$ in (A-4) by its equivalent in (A-5) yields $\frac{\partial q_1}{\partial K_1}$, as shown in (A-6):

$$\begin{aligned}\frac{\partial q_1}{\partial K_1}\left(1-\frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)&=\frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial K_1}+\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\frac{\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial K_1}+\frac{\partial D_1}{\partial K_1}}{1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}\\&\Rightarrow\frac{\partial q_1}{\partial K_1}\underbrace{\left[\left(1-\frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)\left(1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right)-\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right]}_{H}=\left(1-\frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right)\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial K_1}\right)+\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial K_1}\\&\Rightarrow\frac{\partial q_1}{\partial K_1}=\frac{\frac{\partial q_1}{\partial \rho_1}-\frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2}-\frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)}{H}\frac{\partial D_1}{\partial K_1}\overset{\text{(A-3)}}{\Rightarrow}\frac{\partial q_1}{\partial K_1}=\frac{\partial q_1}{\partial P_1}\frac{\partial D_1}{\partial K_1}\end{aligned} \tag{A-6}$$

Then, plugging (A-6) into (A-5) yields $\frac{\partial q_2}{\partial K_1}$. $\frac{\partial q_1}{\partial K_2}$ and $\frac{\partial q_2}{\partial K_2}$ can be similarly derived. □

Proof of Eqs. (7) and (8): The first-order conditions for the social planner are as follows:

$$\frac{\partial SW}{\partial P_1} = \frac{\partial U}{\partial q_1}\frac{\partial q_1}{\partial P_1} + \frac{\partial U}{\partial q_2}\frac{\partial q_2}{\partial P_1} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_1} + \left(h_2 + v_2 - c_2 - D_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial P_1} = 0 \tag{A-7}$$

$$\frac{\partial SW}{\partial P_2} = \frac{\partial U}{\partial q_1}\frac{\partial q_1}{\partial P_2} + \frac{\partial U}{\partial q_2}\frac{\partial q_2}{\partial P_2} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_2} + \left(h_2 + v_2 - c_2 - D_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial P_2} = 0 \tag{A-8}$$

$$\begin{aligned}\frac{\partial SW}{\partial K_1} &= \frac{\partial U}{\partial q_1}\frac{\partial q_1}{\partial K_1} + \frac{\partial U}{\partial q_2}\frac{\partial q_2}{\partial K_1} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial K_1} + \left(h_2 + v_2 - c_2 - D_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial K_1} - q_1\frac{\partial D_1}{\partial K_1}\\ &\quad - r_1\\ &= 0\end{aligned} \tag{A-9}$$

$$\frac{\partial SW}{\partial K_2} = \frac{\partial U}{\partial q_1}\frac{\partial q_1}{\partial K_2} + \frac{\partial U}{\partial q_2}\frac{\partial q_2}{\partial K_2} + \left(h_1 + v_1 - c_1 - D_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial K_2} + \left(h_2 + v_2 - c_2 - D_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial K_2} - q_2\frac{\partial D_2}{\partial K_2} - r_2 = 0 \tag{A-10}$$

By replacing the marginal effects of capacities on demands in (A-9) and (A-10) by their equivalents in (6.1) and (6.2), the socially optimal capacity rule in (7) will be obtained. It should be noted, using (6.1) and (6.2), that (A-9) and (A-10) reduce to $\frac{\partial SW}{\partial K_1} = \frac{\partial D_1}{\partial K_1}\left(\frac{\partial SW}{\partial P_1} - q_1\right) = r_1$ and $\frac{\partial SW}{\partial K_2} = \frac{\partial D_2}{\partial K_2}\left(\frac{\partial SW}{\partial P_2} - q_2\right) = r_2$, which then yield (7) by recalling from (A-7) and (A-8) that $\frac{\partial SW}{\partial P_1} = \frac{\partial SW}{\partial P_2} = 0$. Plug $\frac{\partial U}{\partial q_1} \equiv \rho_1$ and $\frac{\partial U}{\partial q_2} \equiv \rho_2$ into (A-7) and (A-8). Noting that $P_i = \rho_i - D_i(q_i, K_i),\ i = 1, 2$, we multiply (A-7) and (A-8) by $\frac{\partial q_2}{\partial P_2}$ and $\frac{\partial q_2}{\partial P_1}$, respectively, and obtain:

$$P_1\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} + P_2\frac{\partial q_2}{\partial P_1}\frac{\partial q_2}{\partial P_2} + \left(h_1 + v_1 - c_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} + \left(h_2 + v_2 - c_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial P_1}\frac{\partial q_2}{\partial P_2} = 0 \tag{A-11}$$

$$P_1\frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1} + P_2\frac{\partial q_2}{\partial P_2}\frac{\partial q_2}{\partial P_1} + \left(h_1 + v_1 - c_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1} + \left(h_2 + v_2 - c_2 - q_2\frac{\partial D_2}{\partial q_2}\right)\frac{\partial q_2}{\partial P_2}\frac{\partial q_2}{\partial P_1} = 0 \tag{A-12}$$

We subtract (A-12) from (A-11):

$$\left(P_1 + h_1 + v_1 - c_1 - q_1\frac{\partial D_1}{\partial q_1}\right)\left(\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} - \frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1}\right) = 0 \tag{A-13}$$

which implies that $P_1^* = c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1}$ (Eq. (8)). Expression for P_2^* can be similarly derived. □

Proof of Eq. (13): Looking into (11) P_1 can be derived as follows:

$$\begin{aligned}&\frac{\partial SW_1}{\partial P_1} = \rho_1\frac{\partial q_1}{\partial P_1} + \frac{\partial\int_0^{q_1}\rho_1(y, q_2)dy}{\partial q_2}\frac{\partial q_2}{\partial P_1} + (h_1 + v_1 - c_1 - D_1)\frac{\partial q_1}{\partial P_1} - q_1\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial P_1} = 0\\ &\overset{(2)}{\Rightarrow} P_1 = c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1} - \frac{\partial\int_0^{q_1}\rho_1(y, q_2)dy}{\partial q_2}\frac{\frac{\partial q_2}{\partial P_1}}{\frac{\partial q_1}{\partial P_1}}\end{aligned} \tag{A-14}$$

Replacing $\frac{\partial q_1}{\partial P_1}$ and $\frac{\partial q_2}{\partial P_1}$ by their equivalents from (5.1) and (5.2) and also using the definition of elasticity, i.e. $\varepsilon_i = \frac{\partial q_i}{\partial \rho_i}\frac{\rho_i}{q_i}$ $(i = 1, 2)$ as price elasticity of demand and $\varepsilon_{ij} = \frac{\partial q_i}{\partial \rho_j}\frac{\rho_j}{q_i}$ $(i, j = 1, 2)$ as cross-price elasticity of demand, (A-14) can be written as (A-15) below:

$$\begin{aligned}P_1 &= c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1} - \frac{\partial\int_0^{q_1}\rho_1(y, q_2)dy}{\partial q_2}\\ &\quad\times\frac{\left(\frac{\partial q_2}{q_2}\frac{\rho_1}{\partial\rho_1}\right)\frac{q_2}{\rho_1}}{\left(\frac{\partial q_1}{q_1}\frac{\rho_1}{\partial\rho_1}\right)\frac{q_1}{\rho_1} - \frac{\partial D_2}{\partial q_2}\left[\left(\frac{\partial q_1}{q_1}\frac{\rho_1}{\partial\rho_1}\right)\frac{q_1}{\rho_1}\left(\frac{\partial q_2}{q_2}\frac{\rho_2}{\partial\rho_2}\right)\frac{q_2}{\rho_2} - \left(\frac{\partial q_1}{q_1}\frac{\rho_2}{\partial\rho_2}\right)\frac{q_1}{\rho_2}\left(\frac{\partial q_2}{q_2}\frac{\rho_1}{\partial\rho_1}\right)\frac{q_2}{\rho_1}\right]}\\ &= c_1 - h_1 - v_1 + q_1\frac{\partial D_1}{\partial q_1} - \frac{\partial\int_0^{q_1}\rho_1(y, q_2)dy}{\partial q_2}\frac{\varepsilon_{21}\rho_2 q_2}{\varepsilon_1\rho_2 q_1 - (\varepsilon_1\varepsilon_2 - \varepsilon_{12}\varepsilon_{21})q_1 q_2\frac{\partial D_2}{\partial q_2}}\end{aligned} \tag{A-15}$$

Proof of Eq. (14): Rearranging (12) P_2 can be expressed as:

$$P_2 = c_2 - h_2 - \frac{q_2}{\frac{\partial q_2}{\partial P_2}} \tag{A-16}$$

Substituting (5.1) for $\frac{\partial q_2}{\partial P_2}$, we obtain

$$
\begin{aligned}
P_2 &= c_2 - h_2 - q_2 \frac{\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)\left(1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right) - \frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}{\frac{\partial q_2}{\partial \rho_2}\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) + \frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}} \\
&= c_2 - h_2 - q_2 \frac{\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) - \frac{\partial D_2}{\partial q_2}\left[\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)\frac{\partial q_2}{\partial \rho_2} + \frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}\right]}{\frac{\partial q_2}{\partial \rho_2}\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) + \frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}} = c_2 - h_2 - q_2\left[-\frac{\partial D_2}{\partial q_2} + \underbrace{\frac{1}{\frac{\partial q_2}{\partial \rho_2} + \frac{\frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}}{1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}}}}_{I}\right] \\
&= \left(c_2 - h_2 + q_2\frac{\partial D_2}{\partial q_2}\right) - \frac{1}{\left(\frac{\partial q_2}{q_2}\frac{\rho_2}{\partial \rho_2}\right)\frac{1}{\rho_2} + \frac{\left(\frac{\partial q_2}{q_2}\frac{\rho_1}{\partial \rho_1}\right)\frac{1}{\rho_1}\frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{q_1}\frac{\rho_2}{\partial \rho_2}\right)\frac{q_1}{\rho_2}}{1 - \left(\frac{\partial q_1}{q_1}\frac{\rho_1}{\partial \rho_1}\right)\frac{q_1}{\rho_1}\frac{\partial D_1}{\partial q_1}}} = \left(c_2 - h_2 + q_2\frac{\partial D_2}{\partial q_2}\right) - \frac{1}{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}}
\end{aligned}
\tag{A-17}
$$

In order to realize the sign of the last term in (A-17), we define I as:

$$
I = \frac{1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}}{\frac{\partial q_2}{\partial \rho_2}\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) + \frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial q_1}{\partial \rho_2}} = \frac{\overbrace{1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}}^{+}}{\underbrace{\frac{\partial q_2}{\partial \rho_2} - \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)}_{-}} < 0 \tag{A-18}
$$

The negative sign of I can be derived as follows. The numerator $(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1})$ is positive since $\frac{\partial q_1}{\partial \rho_1} < 0$ and $\frac{\partial D_1}{\partial q_1} > 0$. On the denominator, $\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1} > 0$ as an airport's own elasticity is always greater than cross elasticity (i.e. $\frac{\partial q_1}{\partial \rho_1}\frac{\rho_1}{q_1} > \frac{\partial q_1}{\partial \rho_2}\frac{\rho_2}{q_1}; \frac{\partial q_2}{\partial \rho_2}\frac{\rho_2}{q_2} > \frac{\partial q_2}{\partial \rho_1}\frac{\rho_1}{q_2}$). On the other hand, $\frac{\partial D_1}{\partial q_1} > 0$ and $\frac{\partial q_2}{\partial \rho_2} < 0$. Therefore, the denominator is negative, and I is negative. Multiplying I by q_2 yields

$$
q_2 I = \frac{1}{\frac{\varepsilon_2}{\rho_2} + \frac{\varepsilon_{12}\varepsilon_{21}}{\rho_1\rho_2}\frac{q_1\frac{\partial D_1}{\partial q_1}}{1 - \frac{\varepsilon_1}{\rho_1}q_1\frac{\partial D_1}{\partial q_1}}} < 0 \tag{A-19}
$$

which is the last term in Eq. (14)

Proof of Eq. (19): When service prices are determined conditional on capacities, the marginal effect of capacity improvement at airport 1 on its demand, i.e. $\frac{dq_1}{dK_1}$, is derived as follows.

$$
\begin{aligned}
&\frac{dq_1}{dK_1} = \frac{dq_1(\rho_1,\rho_2)}{dK_1} = \frac{\partial q_1}{\partial \rho_1}\frac{\partial \rho_1}{\partial K_1} + \frac{\partial q_1}{\partial \rho_2}\frac{\partial \rho_2}{\partial K_1} = \frac{\partial q_1}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\frac{dq_1}{dK_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_1}{\partial \rho_2}\left(\frac{\partial P_2^*}{\partial K_1} + \frac{\partial D_2}{\partial q_2}\frac{dq_2}{dK_1}\right) \\
&\Rightarrow \frac{dq_1}{dK_1}\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) = \frac{\partial q_1}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_1}{\partial \rho_2}\left(\frac{\partial P_2^*}{\partial K_1} + \frac{\partial D_2}{\partial q_2}\frac{dq_2}{dK_1}\right)
\end{aligned}
\tag{A-20}
$$

$$
\begin{aligned}
&\frac{dq_2}{dK_1} = \frac{dq_2(\rho_1,\rho_2)}{dK_1} = \frac{\partial q_2}{\partial \rho_1}\frac{\partial \rho_1}{\partial K_1} + \frac{\partial q_2}{\partial \rho_2}\frac{\partial \rho_2}{\partial K_1} = \frac{\partial q_2}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\frac{dq_1}{dK_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_2}{\partial \rho_2}\left(\frac{\partial P_2^*}{\partial K_1} + \frac{\partial D_2}{\partial q_2}\frac{dq_2}{dK_1}\right) \\
&\Rightarrow \frac{dq_2}{dK_1}\left(1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right) = \frac{\partial q_2}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\frac{dq_1}{dK_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_2}{\partial \rho_2}\frac{\partial P_2^*}{\partial K_1}
\end{aligned}
\tag{A-21}
$$

Replacing $\frac{dq_2}{dK_1}$ in (A-20) by its equivalent from (A-21), $\frac{dq_1}{dK_1}$ is derived as follows:

$$
\begin{aligned}
&\frac{dq_1}{dK_1}\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right) = \frac{\partial q_1}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_1}{\partial \rho_2}\left(\frac{\partial P_2^*}{\partial K_1} + \frac{\partial D_2}{\partial q_2}\frac{\frac{\partial q_2}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial q_1}\frac{dq_1}{dK_1} + \frac{D_1}{\partial K_1}\right) + \frac{\partial q_2}{\partial \rho_2}\frac{\partial P_2^*}{\partial K_1}}{1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}}\right) \\
&\Rightarrow \frac{dq_1}{dK_1}\underbrace{\left[\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\right)\left(1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right) - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\frac{\partial D_2}{\partial q_2}\right]}_{H} \\
&= \frac{\partial q_1}{\partial \rho_1}\left(1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right)\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_1}{\partial \rho_2}\frac{\partial P_2^*}{\partial K_1}\left(1 - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\right) + \frac{\partial q_1}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\left[\left(\frac{\partial q_2}{\partial \rho_1}\left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial K_1}\right) + \frac{\partial q_2}{\partial \rho_2}\frac{\partial P_2^*}{\partial K_1}\right)\right] \\
&\Rightarrow \frac{dq_1}{dK_1} = \frac{\frac{\partial P_2^*}{\partial K_1}\frac{\partial q_1}{\partial \rho_2} + \left(\frac{\partial P_1^*}{\partial K_1} + \frac{\partial D_1}{\partial K_1}\right)\left[\frac{\partial q_1}{\partial \rho_1} - \frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)\right]}{H} \\
&\overset{\text{(A-6)}}{\Rightarrow} \frac{dq_1}{dK_1} = \frac{\partial q_1}{\partial K_1} + \frac{\frac{\partial P_2^*}{\partial K_1}\frac{\partial q_1}{\partial \rho_2} + \frac{\partial P_1^*}{\partial K_1}\left(\frac{\partial q_1}{\partial \rho_1} - \frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)\right)}{H}
\end{aligned}
\tag{A-22}
$$

$\frac{dq_2}{dK_1}$ can be similarly obtained by plugging (A-20) into (A-21). The derivation of $\frac{dq_1}{dK_2}$ and $\frac{dq_2}{dK_2}$ is likewise. □

Proof of Eqs. (44): Multiplying the first order derivatives with respect to P_i, i.e. $\frac{\partial \pi}{\partial P_i} = (P_i + h_i - c_i)\frac{\partial q_i}{\partial P_i} + (P_j + h_j - c_j)\frac{\partial q_j}{\partial P_i} + q_i = 0$, by $\frac{\partial q_j}{\partial P_j}\,(i,j = 1,2)$ and then subtracting them yields, for example, for $i = 1;\ j = 2$:

$$(P_1 + h_1 - c_1)\left(\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} - \frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1}\right) + q_1\frac{\partial q_2}{\partial P_2} - q_2\frac{\partial q_2}{\partial P_1} = 0 \tag{A-23}$$

Then, by replacing $\frac{\partial q_i}{\partial P_j}\ (i,j = 1,2)$ by (5.2), $\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} - \frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1}$ is simplified as follows:

$$\begin{aligned}\frac{\partial q_1}{\partial P_1}\frac{\partial q_2}{\partial P_2} - \frac{\partial q_1}{\partial P_2}\frac{\partial q_2}{\partial P_1} &= \frac{1}{H^2}\left[\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right) - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)\right.\\ &\quad \left. + \frac{\partial D_1}{\partial q_1}\frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)^2 - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right] = \frac{1}{H^2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)\\ &\quad \times \underbrace{\left(1 - \frac{\partial q_1}{\partial \rho_1}\frac{\partial D_1}{\partial q_1} - \frac{\partial q_2}{\partial \rho_2}\frac{\partial D_2}{\partial q_2} + \frac{\partial D_1}{\partial q_1}\frac{\partial D_2}{\partial q_2}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)\right)}_{H} = \frac{\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}}{H}\end{aligned} \tag{A-24}$$

Plugging (A-24) into (A-23) and replacing $\frac{\partial q_2}{\partial P_2}$ and $\frac{\partial q_2}{\partial P_1}$ by (5.1) and (5.2) yield:

$$P_1^* = c_1 - h_1 - q_1\frac{\frac{\partial q_2}{\partial \rho_2} - \frac{\partial D_1}{\partial q_1}\left(\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}\right)}{\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}} + q_2\frac{\frac{\partial q_2}{\partial \rho_1}}{\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}} = c_1 - h_1 + q_1\frac{\partial D_1}{\partial q_1} + \frac{-q_1\frac{\partial q_2}{\partial \rho_2} + q_2\frac{\partial q_2}{\partial \rho_1}}{\frac{\partial q_1}{\partial \rho_1}\frac{\partial q_2}{\partial \rho_2} - \frac{\partial q_1}{\partial \rho_2}\frac{\partial q_2}{\partial \rho_1}} \tag{A-25}$$

Further noting that $\frac{\partial q_i}{\partial \rho_i} = \frac{\varepsilon_i q_i}{\rho_i}$ and $\frac{\partial q_i}{\partial \rho_j} = \frac{\varepsilon_{ij} q_i}{\rho_j}$, Eq. (44) is obtained. Expression for P_2^* can be similarly derived. □

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